PART II: Music & Mathematics

AGE RANGE: 16-18

TOOL 25: BACH AND THE MUSICAL MOEBIUS STRIP

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Educator’s Guide

Title: Bach and the musical Moebius strip

Age Range: 16-18 years old

Duration: 2 hours

Mathematical Concepts: Infinity, circumference of a circle, radius and diameter of a circle, Two-dimensional vs three-dimensional.

Artistic Concepts: Two-dimensional vs three-dimensional. Handicraft.

General Objectives: This is a great tool for letting pupils create and at the same time discover a classic image of art.

Instructions and Methodologies: Give the students the possibility to explore math through music and handicraft, by applying it to hands-on doing. This tool is a good basis for your class to discover different math concepts by actually working with their hands.

Resources: This tool provides pictures and videos for the educator to use in the classroom. The topics addressed in these resources will also be an inspiration to find other materials that might be relevant in order to personalize and give nuance to the lesson.

Tips for the educator: Even though there are many hands-on activities involved, remember to be exact about the mathematics.

Desirable Outcomes and Competences: At the end of this tool, the student will be able to:
- Understand infinity in an improved way.
- Explore their handicraft skills.

Debriefing and Evaluation:

Write 3 aspects you liked about this activity:	1. 2. 3.
Write 2 aspects that you have learned	1. 2.
Write 1 aspect for improvement	1.

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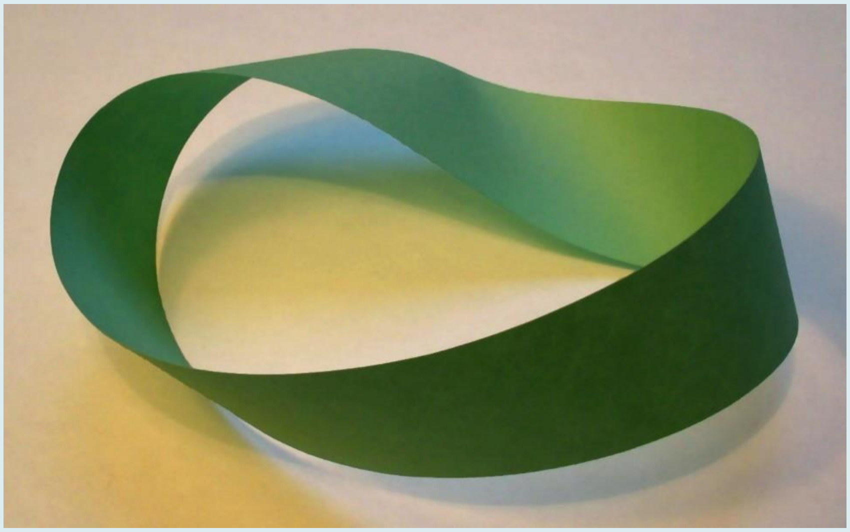
Introduction

Is the Moebius band rediscovered by August Ferdinand Moebius or was it Moebius who discovered it? Already the ancient Greeks used the symbol Moebius so carefully studied, to denote eternity and infinity. Moebius, on the other hand, discovered the mathematical properties of the band, that is, it has one side and one edge.



Picture Möbius strip https://commons.wikimedia.org/wiki/File:M%C3%B6bius_strip.jpg

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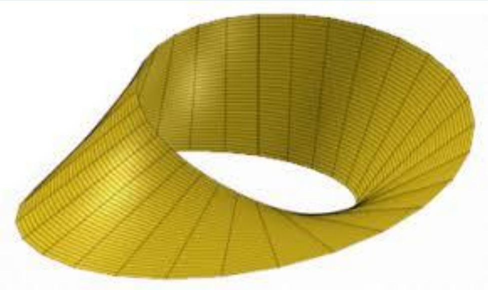
The Moebius strip

August Ferdinand Moebius, born November 17, 1790 in Schulpforta, died September 26, 1868 in Leipzig, was a German mathematician and astronomer.

In 1816, Moebius became extra ordinary professor of astronomy and in 1844 professor of higher mechanics and astronomy at the University of Leipzig. His main research work belongs to the pure mathematics, where he invented a new geometric method, the so-called barycentric calculation. Barycentric calculations use barycentric coordinates .

His most famous result is the so-called Moebius strip, which is a non-orientable surface that only has one side. While Moebius was fully engaged in thinking about how this strip could be used in different ways, at the same time another researcher called Listing was on the same paths about a two-dimensional strip that only has one side and one edge. The two scientists simultaneously published articles about the strip's functions and came to the same result roughly at the same time but it was Moebius name that was finally used to name the strip - and the world now had the Moebius Strip.

The Moebius strip:



Picture 2: The Moebius strip https://commons.wikimedia.org/wiki/File:M%C3%B6bius_strip_(plot).png

- is a long rectangular surface
- which is rotated 180 degrees with the ends assembled

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- so that along its new path it has one side and one border.
- The surface is non-orientable and comes back to the same point all the time but mirrored as it only has one side.

Not only Listing and Moebius had been fascinated by the single-sided strip where a dash can be painted on "all" pages without the pen being lifted. Even today, the Moebius strip is used in graphic design as it creates a dynamic and unlimited image. Within science fiction literature, the Moebius strip is used as a description for a possible universe.

The Moebius strip is the mathematical object most used outside the world of mathematics. Comparing the Moebius strip made from paper to a music loop, you will learn that a piece of music which can be played through from the beginning to the end sounding harmonically and melodically correct (basically, sounding nice) is the same as going around the Moebius strip once. Then, if you go through it a second time, yet you start at the end of the piece, so that the last note becomes the first note of the piece, and still it sounds nice, you have Moebius music. To find out this to yourself, print the notes, cut them out, glue them into Moebius strips.

Have a look at a musical Moebius strip here: https://www.youtube.com/watch?v=3x03nJnk-wk

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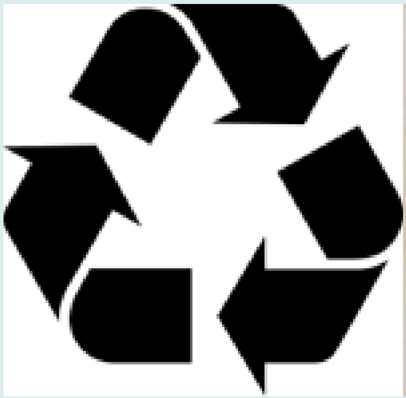


Picture 3: Recycle symbol https://creazilla.com/nodes/46010-recycling-symbol-emoji-clipart
Picture 4: Möbius wedding band
https://commons.wikimedia.org/wiki/File:M%C3%B6biusWeddingBand.JPG

The current use of Moebius strips include, among other things, the conveyor belt that is found in the checkout line in a super market. The actual band that transports the goods we buy is shaped like a Möbius strip, as this reduces wear and thus increases the service life. During the early industrialization period, Moebius strip was used as the link between steam engines and the machines that steam engines operated (lathes, threshers, etc.)

Picture 5: Handmade luxury infinity hood
https://www.flickr.com/photos/smittenkittenoriginals/5080610523/

You can make your own Moebius strip by taking a rectangular paper strip, turning one end half a turn and sticking the ends together. If you now think that someone,

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say an ant, crawls along the strip, when it crawls, it will be on the other side of the band. Thus, the Moebius band has a single side. Moebius discovered the band as he watched triangulations of the plane.

Glossary

Barycentric coordinates: In astronomy, barycentric coordinates are non-rotating coordinates with the origin at the barycenter of two or more bodies.

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The Math behind The Moebius strip

A Moebius strip can be similar to an area in a coordinate system. This part of the tool will show how this area can be calculated using integrals and functions.

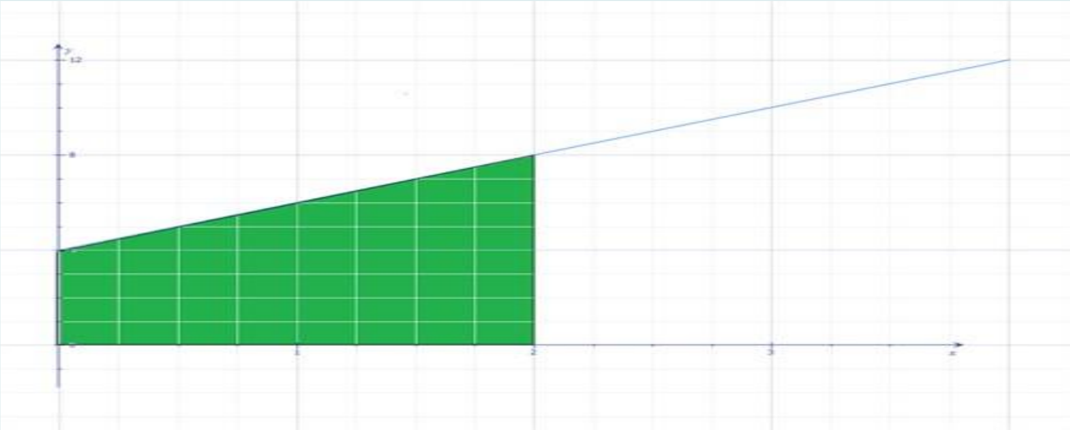
Integrals and functions

When calculating the integral of a function it is equivalent to calculating the area between the graph and the x-axis.

Let's start with an example

We have the following function

$$f(x) = 2x + 4$$



Picture 6: Coordinate system

and are interested in knowing the area of the area that lies between the graph and the x-axis, and which is bounded by the vertical lines x = 0 and x = 2.

There is a general formula for calculating these types of areas:

$$\int_a^b f(x)\,\mathbf{dx} = \mathbf{F\,(b)} - \mathbf{F\,(a)}$$

In the left, we first have the integral sign

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∫

The numbers a and b indicate the lower and upper bound for the area we are interested in (in our example, a = 0 and b = 2). To the right of the integral sign with its boundaries comes the function that constitutes the upper bound of the area. At the last left, comes dx, which indicates that the area calculation must be done with respect to change in the x-joint.

In the right-hand corner the difference is indicated

$$\mathbf{F}\,(\mathbf{b})\;-\mathbf{F}\,(\mathbf{a})$$

This is the difference between the value of the primitive function F at the upper limit (x = b) and the lower limit (x = a).

It is very new that comes with this formula at once, so the simplest thing is to continue with our example:

So we have the known function y (x) = 2x + 4 and we know the lower limit at a = 0 and the upper limit at b = 2. Thus, we can set up the left-hand part of the formula, which in our example looks like this (we explicitly assume in this case that the integral is an area, A):

$$A = \int_0^2 (2x + 4)\,dx$$

The right-hand part of the formula includes the primitive function F, which we do not know yet, so in the next step we have to calculate it, which we do based on the rules we came up with in the previous section. We get the following:

$$F\,(x) \;=\; x^2 + 4x \;+\; C$$

When calculating the integral, one usually writes the calculation as follows:

$$\int_a^b f(x)\,dx \;=\; [F\,(x)]_a^b$$

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which in our example becomes

$$\int_0^2 (2x+4)\,\boldsymbol{dx} = [\boldsymbol{x^2} + \boldsymbol{4x}]_0^2$$

As you may have noticed in the formula above, we ignored the constant term C as we printed the right-hand side. The reason for this is that this term will disappear as it is included in both F (b) and F (a). However, we include the constant term in the following calculation, so that you see how it disappears:

$$\boldsymbol{A = \int_0^2 (2x+4)\,dx = [x^2 + 4x]_0^2 =}$$

$$\boldsymbol{= (2 \cdot 2 + 4 \cdot 2 + C) - (0 \cdot 2 + 4 \cdot 0 + C) =}$$

$$\boldsymbol{= 4 + 8 + C - C = 12\,a.e.}$$

Thus, the area searched is 12 area units.

In the example above, we had a function whose graph over the entire interval was above the x-axis. Thus, the entire area we would calculate the area on is above the x-axis.

What happens to our calculations if the function should have negative values in the interval and the area calculated in that case is below the x-axis? Well, then integration calculations based on the method we used above will lead to a negative result. But an area cannot have a negative value, which is why we have to change the sign of the integral if the area to calculate the area is below the x-axis

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TASK

Area calculation of a limited area

Calculate the area of the area bounded by the curve $Y = x^2 + 2x + 2$ the x-axis and

a) The lines x = -1 and x = 1

b) The lines x = -2 and x = 0

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LEARN MORE…

Ants walking on a Moebius strip

https://www.youtube.com/watch?v=ZN4TxmWK0bE

Two classroom lessons dealing with the Moebius strip

https://www.youtube.com/watch?v=JNtKcK27x1s

https://www.youtube.com/watch?v=1xKiSSVY5bI

A short sci-fi movie containing a Moebius strip theme

https://www.youtube.com/watch?v=HD9MYY0aPug

A short introduction to the line integral

https://www.khanacademy.org/math/multivariable-calculus/integrating-multivariable-functions/line-integrals/v/introduction-to-the-line-integral

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