PART II: Music & Mathematics

AGE RANGE: 13-15

TOOL 15: PYTHAGOREAN TUNING AND RATIOS

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TheArtOf Maths

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Educator's Guide

Title: Pythagorean Tuning and Ratios

Age Range: 13-15 years old

Duration: 1 hour

Mathematical Concepts: Pythagoras tuning, ratios

Artistic Concepts: Notes, Intervals, sound waves, pitch, frequency, Pythagoras

General Objectives: To discover the mathematical concepts hidden in musical compositions and acquire a more practical view of the use of mathematics.

Instructions and Methodologies: The students will explore both fields as a whole, by listening to the music or playing it and watching the suggested videos that analyze musical compositions. They will discover the basis of the mentioned math concepts.

Resources: This tool provides videos and online resources for you to use in your classroom. The topics addressed in the tool will help you find other materials to personalize and give nuance to your lesson.

Tips for the educator: Learning by doing is very efficient, especially for young learners with learning difficulties. Always explain what each math concept is useful for, practically and create a hands-on experience for them.

Desirable Outcomes and Competences: At the end of this tool, the student will be able to:

- Understand the logical process behind music composition;
- Understand the use of Pythagorean tuning in music;
- Understand and calculate the ratios in intervals.

Debriefing and Evaluation:

Write 3 aspects you liked about this activity:	1. 2. 3.
Write 2 aspects that you have learned	1. 2.
Write 1 aspect for improvement	1.

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Introduction

Music and mathematics don't show an obvious connection for those who have never composed or read a music sheet. However, it appears clearly that the timing of musical compositions and the structuring of the sheet by measures evokes a mathematical way of thinking.

Many scholars have studied the implication of mathematics in the arts. Music was one of the focus points of their studies and it was found that, throughout history, many mathematicians had explored that question. Pythagoras, Leonardo Bonacci (also known as Fibonacci), and many others have contributed to the research. Different aspects of mathematics, ranging from basic geometry and number sequences to trigonometry, have shown to be used in musical compositions.

Within this tool, we will focus on the applicability of mathematics in musical compositions by first investigating the Pythagorean tuning system and exploring the options it offers for music composition.

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How does music work?

When we play music, the vibration produced, and the movement of air particles goes through our ears and allow us to hear the sounds at the right frequency. If you look at a guitar string, you can see it move in a certain way and at a certain pace. When we stretch a string, its pitch goes higher and its frequency faster. What is produced is called a sound wave and it goes right into our ears, moving the fluid of our cochlea, in the inner part of our ear.

Of course, Pythagoras, a Greek philosopher from around 570 – c. 495 BC was not aware of everything we know today about the human body and musical composition. However, he developed a theory on how to calculate the ratios in intervals, which is what you will learn in this lesson. The legend says he heard different sounds coming from hammers at the blacksmith's shop and found out that when a hammer was twice as big or heavy as another, it produced the same note one octave higher.

You can learn more about where sounds come from with the following video:

https://www.youtube.com/watch?v=i_0DXxNeaQ0

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Glossary

Frequency: gives us the speed of a vibration and the pitch of a sound.

Pitch: is whether a note sounds high or low and is measured in Hertz.

Sound wave: represents the vibration produced by a sound. Its length and speed determine the pitch or frequency of the sound.

Cochlea the spiral cavity situated in the inner ear that reacts to sound vibrations.

Interval: is the pitch difference between two sounds.

Octave: is the pitch difference between one note and another that has twice its frequency

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The math behind music composition

Watch this video by Working iveshoot which explains this phenomenon with more details: https://www.youtube.com/watch?v=rTT1XHJKKug.

Notes and Intervals:

A tuning system is one that is used to determine which pitches to play.

There are twelve notes in the Pythagorean tuning system:

Figure 1: Piano tiles with notes

The intervals are made of two notes. For instance, the interval C to E is called the major third interval. The Pythagorean tuning is based on the ratio **(1:2)** in intervals called **octaves,** such as C to C' and on the ratio **(2:3)** in intervals called the **perfect fifth intervals**, such as C to G.

Pythagoreans believed that only the notes whose frequencies were a fraction with small numbers were pleasing to hear together. This is why they chose the octave and the perfect fifth as a basis to build musical compositions.

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How does it work?

According to Pythagorean Tuning, to calculate the ratios within a scale, we will need to work with the octave and the fifth perfect interval.

Let's see some examples: starting with C and **using only major notes (the white ones):**[1]

1) What we know: C to G is (2:3) and G to D' is also (2:3)

   What we want: To find C to D'

   What we do: we multiply both ratios.

   The math: $\frac{3}{2} \text{ x } \frac{3}{2} = \frac{9}{4}$

   So, the ratio is (4:9)

C	D	E	F	G	A	B	C'	D'	E'	F'	
2:				3							**x2**
				2:				3			**x3**
4:								**9**			

2) What we know: C to D' is (4:9) and the octave D to D' is (1:2)

   What we want to find: C to D

   What we do: we divide the ratio (4:9) by (1:2)

   The math: $\frac{9/4}{2/1} = \frac{9}{4} \ x \ \frac{1}{2} = \frac{9}{8}$

   So, the ratio is (8:9)

C	D	E	F	G	A	B	C'	D'	E'	F'	
4:								9:			**x2**
	1:							2			**X9**
8:	**9**										

[1] Inspired by a video by Working iveshoot: https://www.youtube.com/watch?v=rTT1XHJKKug

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TASK

Look for more ratios![2]

Find the other elements of the major scale starting with C.

To do so, fill in the following tables.

1) What we know: C to D is (8:9) and D to A is (2:3)

   What we want to find: C to A

   What we do:

   The math:

C	D	E	F	G	A	B	C'	D'	E'	F'	

2) What we know: C to A is (____: ____) and A to E' is (2:3)

What we want to find: C to E'

What we do:

The math:

C	D	E	F	G	A	B	C'	D'	E'	F'	

[2] Inspired by Working iveshoot's video : https://www.youtube.com/watch?v=rTT1XHJKKug

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3) What we know: C to E' is (____: ____) and E to E' is (1:2)

What we want to find: C to E

What we do:

The math:

C	D	E	F	G	A	B	C'	D'	E'	F'	

4) What we know: C to E is (____:____) and E to B is (2:3)

What we want to find: C to B

What we do:

The math:

C	D	E	F	G	A	B	C'	D'	E'	F'	

5) What we know: C to B is (____:____) and B to F' is (2:3)

What we want to find: C to F'

What we do:

The math:

C	D	E	F	G	A	B	C'	D'	E'	F'	

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6) What we know: C to F' is (____:____) and F to F' is (1:2)

What we want to find: C to F

What we do:

The math:

C	D	E	F	G	A	B	C'	D'	E'	F'	

You can now give the ratios for all the notes!

C	D	E	F	G	A	B	C'
1/1	9/8			3/2			2/1

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LEARN MORE…

Video about where sound comes from:

https://www.youtube.com/watch?v=i_0DXxNeaQ0

Video about how math is used in music:

https://www.youtube.com/watch?v=rTT1XHJKKug

Pythagoras' theories in music, geometry and math:

http://www.historyofmusictheory.com/?page_id=20

"An Exploration of the Relationship between Mathematics and Music" by Saloni Shah, 2010:

http://eprints.ma.man.ac.uk/1548/1/covered/MIMS_ep2010_103.pdf

Experiment with Pythagorean Tuning:

https://www.youtube.com/watch?v=CKGsiGYzYxA

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