PART IV: Cinematography & Mathematics

AGE RANGE: 13-15

TOOL 33: PRIME NUMBER THEORY AND PARTITIONS IN 'THE MAN WHO KNEW INFINITY' BY MATTHEW BROWN

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Educator's Guide

Title: Prime Number theory and partitions in 'The man who knew infinity' by Matthew Brown

Age Range: 13-15 years old

Duration: 2 hours

Mathematical Concepts: Prime Number Theory and Partitions

Artistic Concepts: Cinematography.

General Objectives: Students will familiarize with prime numbers and partitions, use the mathematical examples given, to analyze the task and familiarize with the formula of partitions and the Young diagram. They will also see some of the steps that a mathematical research involves. They will have the chance to see the vast number of mathematical movies there are and get acquainted with Ramanujan, his work and his biography, through pictures, videos and some literacy excerpts.

Instructions and Methodologies: The methodologies used here follow Blooms' Taxonomy, starting with the knowledge of who Ramanujan was, a reminder of what prime numbers and partitions are, to a more comprehensive level of explaining/ summarizing them. Then, they will apply the theory learnt through the task given for the ultimate goal of being able to relate this knowledge on partitions.

Resources: This tool provides Youtube videos consisting of a synopsis of the actual life of Srinivasa Ramanujan and excerpts of the film *'The man who knew infinity'*. There are some pictures; a glossary, Ramanujan's biography and the mathematics behind; examples of partitions; the task itself; some additional online resources to explore.

Tips for the educator: It will be important to grasp the interest of your students by emphasizing the difficulties Ramanujan was phasing in his time and real life (including poverty and obstacles due to his Indian origin). It is a good idea to also emphasize on elements of someone's character as an example/role model to students, which helped Ramanujan excel, beyond his prodigy mind, like his persistency, hard work and devotion which ultimately helped him remain in history (these elements are obvious through the video link on his biography).

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Desirable Outcomes and Competences: Students will identify who this great mathematician was (biographical elements); whilst through the task they will experiment with the visual representation of partitions using Young diagram.

Debriefing and Evaluation Questions:

As part of reflection and/or formative assessment (=in order to improve the tool for the next time according to the students' background, interest, exact age, country's culture, students' prior knowledge etc) the educator can use these cards sometimes called EXIT CARDS either by a hard copy he/she has made from before or simply by posing these statements on board and the students write the answers on a paper which they will leave preferably anonymously while exiting the room. The specific formative strategy is called 3,2,1. For more strategies you can visit:

https://www.bhamcityschools.org/cms/lib/AL01001646/Centricity/Domain/131/70%20Formative%20Assessments.pdf

Write 3 things you liked about this activity	1. 2. 3.
Write 2 things you have learned	1. 2.
Write 1 aspect for improvement	1.

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Introduction

According to Polster (2012) there are more than 700 mathematical movies although some related to mathematics to a very big extent and some to a much smaller; they are considered as an injection of moments of fun, which can be used into courses in an attempt to make the learning of mathematics fun and interesting for young audiences. For this task, 'The man who knew infinity' which is based on the homonymous Book by Robert Kanigel, has been chosen for several reasons.

Firstly, it is one of the movies alertly related to mathematics and the story of a great Indian mathematician of the 20th century, called Srinivasa Ramanujan. Also, the film offers great insights presenting mathematics as art but also as a creative process of discovery, lending into several mathematical concepts and mostly, prime numbers and partitions. The movie also sets a good role model for young adults.

The film captivates the grasps of what it means to undertake mathematical research. The protagonist is mostly urged by curiosity and tries to capture the striking and graceful connections among abstract concepts. These explorations naturally involve some sort of experimentation but rely mostly on ideas and symbols instead of physical things. As we can see both in the book and the movie, there are lots of mistakes and dead ends. Thus, more persistency is needed. That is why when the character enters the more typical education of the English University, he is required to give proofs – complete, verifiable, logical justifications– of his assertions. Constructing the proof can be difficult and often takes a lot longer than the initial discovery.

What is emphasized through the film, and an obligation of real mathematical research, is to avoid the temptation even of such great minds to move from discovery to discovery, from one connection to another, before giving the proofs to support the ones already found. Tertiary education in mathematics aims to instill this. In India, Ramanujan lacked such an education. At Cambridge, he had to catch up and fill those gaps. Students, educators and practitioners of mathematics will notice some high-quality practices in this film.

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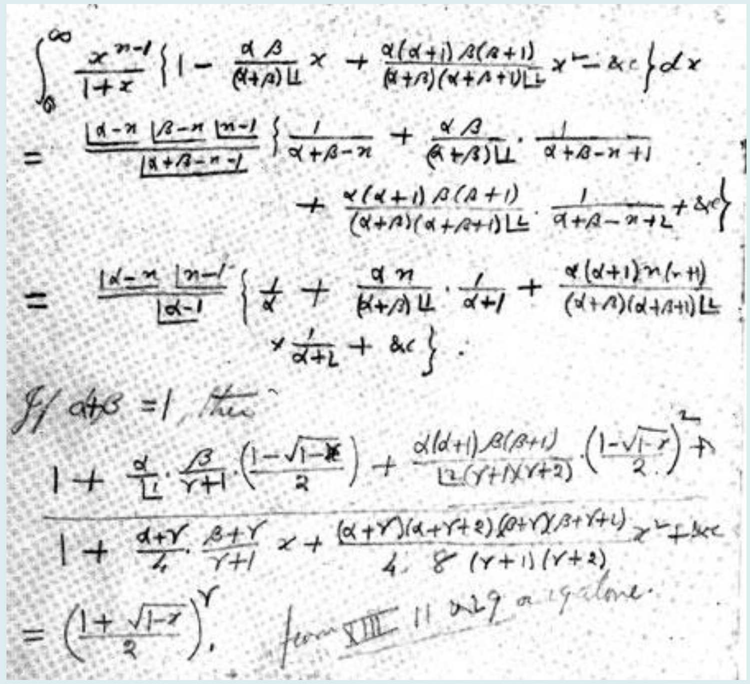
Biography

Picture 1: Original notes of Ramanujan[1]



Picture 2: Ramanujan's notes[2]

[1] Retrieved from: https://www.google.com/search?q=notebooks+of+ramanujan+pdf&client=firefox-b-d&source=lnms&tbm=isch&sa=X&ved=0ahUKEwiZn5uFtd7iAhWNyKQKHVdoD10Q_AUIECgB&biw=1138&bih=527#imgdii=CdWlT6ACYDdArM:&imgrc=dNSzdmvpv-YSrM

[2] Retrieved from: https://www.google.com/search?client=firefox-b-d&biw=1138&bih=527&tbm=isch&sa=1&ei=ABP-XKLMCszLwQL6-ZToCw&q=ramanujan%27s+notebooks&oq=ramanujan%27s+notebooks&gs_l=img.3.0.0j0i5i30j0i8i30j0i24l2.39595.44881..50313...0.0..0.154.2622.0j21......0....1..gws-wiz-img.......35i39j0i67j0i30.GMm5Q9WIy7M#imgrc=omvjbPsONG-PZM

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Srinivasa Ramanujan (22 December 1887 – 26 April 1920) [3]

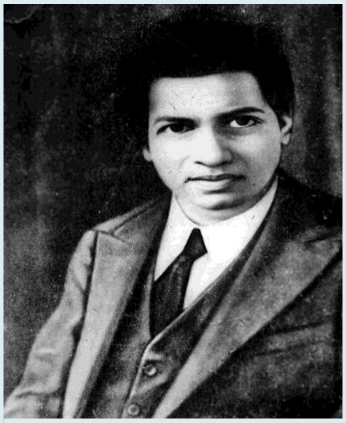

Figure 3: Picture of Srinivasa Ramanujan

Srinivasa Ramanujan was an Indian mathematician and lived during British colonial rule in India. Though he had almost no formal training in mathematics, he made significant contributions to mathematical analysis, number theory, infinite series, and continued fractions, including solutions to mathematical problems then considered to be unsolvable. He started developing his own research, but other professional mathematicians were not interested as his findings were too innovative and presented unusually. In 1913, He wrote to the English mathematician G.H. Hardy at the University of Cambridge, in England. Fascinated by the samples that Ramanjuan had sent him, Hardy arranged for him to travel to Cambridge. Ramanujan had created groundbreaking theorems, among which Hardy found himself and his colleagues outperformed by those new theorems and his rediscovery of recently proven but highly advanced results.

Among his original works, we can find the Ramanujan prime and theta function, which have transformed the world of mathematics and the opportunities it offers and were almost all proven right. Many mathematicians influenced by Ramanujan have published their work in The Ramanujan Journal, a peer-reviewed scientific journal. His notes (published or not) have been examined since he died to elaborate new theories.

In 1919, his health made him return to India, where he died in 1920 at the age of 32. His last letters to Hardy, written January 1920, show that he was still producing new mathematical ideas and theorems.

His life: https://www.youtube.com/watch?v=P0idBBhGNgU

[3] Retrieved from: https://en.wikipedia.org/wiki/Srinivasa_Ramanujan

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Plot of the movie 'The man who knew infinity'

At the 20th century, Srinivasa Ramanujan is an under pressure and poor citizen living in Madras, in India, working at unskilled jobs on the verge of poverty. His employers notice his exceptional skills in mathematics and start giving him basic tasks in accounting. When they realize that his mathematical insights exceed the simple accounting tasks, they encourage him to make his own writings in mathematics available to the public and to contact mathematics professors at universities outside of India. One of those letters is sent to G.H. Hardy, a famous mathematician at the University of Cambridge, who takes particular interest in Ramanujan.

Ramanujan gets married while working and sending out his first publications. Hardy almost immediately invites him to Cambridge to assess his determination as a possible theoretical mathematician. Ramanujan is excited by the opportunity and decides to chase Hardy's offer, even though this means parting from his wife for a long period. He parts with her and promises to keep writing her letters.

As soon as he arrives to Cambridge, Ramanujan faces various forms of racial chauvinism and finds the alteration to life in England more difficult than expected. Although Hardy is very impressed by Ramanujan's abilities, he is worried about his lack of experience in writing proofs, but with determination he manages to get Ramanujan published in a major journal. In the meantime, Ramanujan finds out that he is suffering from tuberculosis whilst his regular letters to his wife remain unanswered after many months. Hardy remains unaware of the personal difficulties he is facing. Ramanujan's health worsens as he continues delving into deeper and more profound research interests in mathematics under the supervision of Hardy and other colleagues at Cambridge.

His wife eventually discovers that his mother has been hiding his letters and not sending hers to him. Hardy tries to get Ramanujan's recognizably exceptional mathematical skills to be totally accepted by his university through nominating Ramanujan for a fellowship at Trinity College. At first, Hardy fails due to politics related

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to the college and the persistent racial prejudice of the time. Later, though by gaining the support of key members of the college, Hardy once more nominates Ramanujan for a fellowship; and he is finally accepted as a Fellow of the Royal Society and later, of Trinity College. In the end, Ramanujan is reunited with his family in India, though his waning health, which suffered mostly from poor housing and harsh winter weather in England, ultimately leads to his premature death soon after his recognition as a mathematician of international merit and importance.

Excerpt from the movie:

Official trailer:
https://www.youtube.com/watch?time_continue=146&v=oXGm9VIfx4w

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Glossary

Number theory: a branch of mathematics that focuses on integers, their properties and relationships.

Infinite series: is a sequence composed of the addition of numbers and which goes on infinitely.

Continued fractions: fractions whose denominator is the sum of an integer with a fraction whose denominator is also the sum of an integer with a fraction, and so on.

Hinduism: is a religion mostly present in Southeast Asia which gives its own idea of spirituality and traditions and is sometimes also considered merely as a "way of life".

Chauvinism: strong patriotism which leads to the belief of a dominant nationality, all others being considered as inferior.

Fellowship: is the status given to someone (the fellow) who is part of a college or society.

College: a group of people who share interests, objectives and professional expertise.

Society: an association of people who share the same goal and/or activities.

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The math behind the man who knew infinity

Prime numbers

A prime number is a whole number greater than 1 whose only factors are 1 and itself. The first few prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23 and 29.

- **Factors** are numbers that we multiply to get another number, for example: if you take the number 6, you see that 2 and 3 are some of its factors.
- Numbers that have more than two factors are called **composite numbers**.
- The number 1 is neither prime nor composite.

Partitions

In number theory and combinatorics, a partition of a positive integer n, also called an integer partition, is a way of writing n as a sum of positive integers.

Two sums that differ only in the order of their summands are considered the same partition. (If order matters, the sum becomes a composition.) A summand in a partition is also called a part.

Ex: 2 + 1 + 3 = 3 + 2 + 1

The number of partitions of n is given by the partition function p(n). The notation $\lambda \vdash n$ means that λ is a partition of n.

Ex: the partition of 4; p(4) = 5

Visual Representation:

Partitions can be graphically visualized with Young diagrams or Ferrers diagrams. They occur in a number of branches of mathematics and physics, including the study of symmetric polynomials and of the symmetric group and in group representation theory in general.

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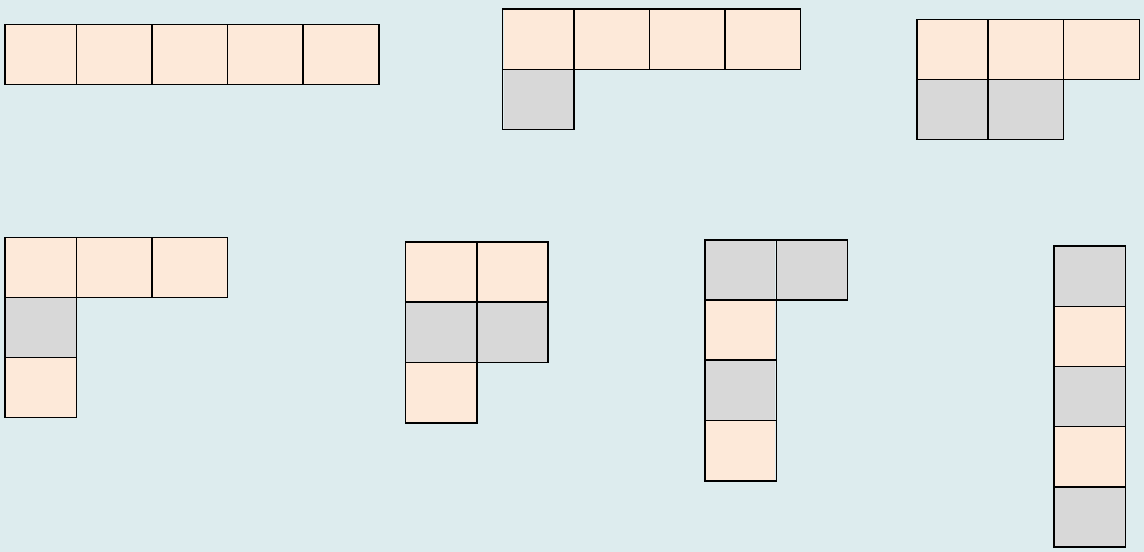
Example

The seven partitions of 5 are:

- 5
- 4 + 1
- 3 + 2
- 3 + 1 + 1
- 2 + 2 + 1
- 2 + 1 + 1 + 1
- 1 + 1 + 1 + 1 + 1

Let's use a Young Diagram to represent the possible partitions of 5:



In some sources, partitions are treated as the sequence of summands, rather than as an expression with "plus" signs. For example, the partition 2 + 2 + 1 might instead be written as the tuple (2, 2, 1) or in the even more compact form (22, 1) where the superscript indicates the number of repetitions of a term.

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TASK

The partition function p(n) represents the number of possible partitions of a positive integer. For instance p(4)=5, because the integer 4 has the five partitions.

Write them all below:

Draw the Young Diagram for each one of the partitions of 4:

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LEARN MORE…

If you want to further investigate on the topics addressed in this tool, you may go through the following links:

Book on maths related to movies:

Polster, B., & Ross, M. (2012). *Math goes to the movies*. Baltimore: Johns Hopkins University Press. Retrieved from http://search.ebscohost.com/login.aspx?direct=true&AuthType=ip,sso&db=nlebk&AN=597694&site=eds-live&custid=s1098328

Ramanujan's Biography:

https://en.wikipedia.org/wiki/Srinivasa_Ramanujan

What is a prime number:

https://whatis.techtarget.com/definition/prime-number

What are partitions:

https://en.wikipedia.org/wiki/Partition_(number_theory)

Ramanujan's Formula:

https://www.newscientist.com/article/dn20039-deep-meaning-in-ramanujans-simple-pattern/

The Man Who Knew Infinity: inspiration, rigour and the art of mathematics

May 24, 2016:

https://theconversation.com/the-man-who-knew-infinity-inspiration-rigour-and-the-art-of-mathematics-59520

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