

$2 < x < 6$

$2 < x < 6$

$2 < y <$

$2 < y <$

$4 < x <$

$4 < x <$

$4 < y <$

$4 < y <$

$2 < x <$

$4 \{2 < x$

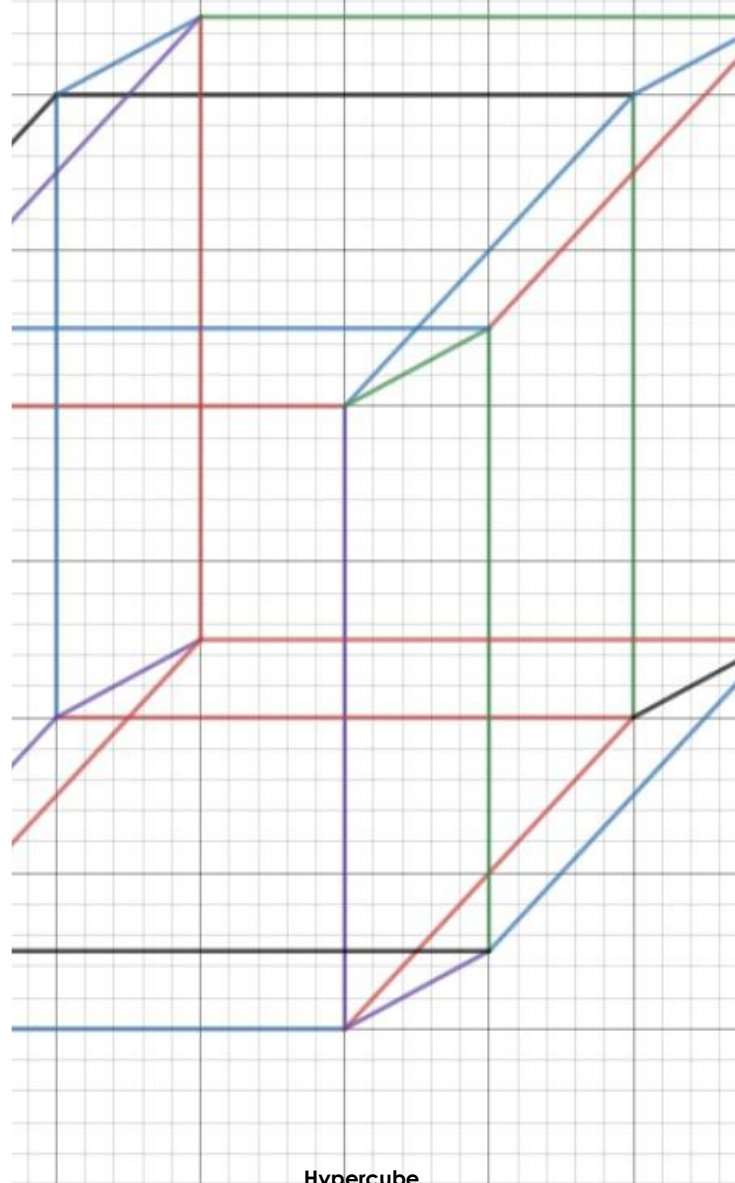
$-4 \{6 <$

$3 < x <$

$+1 \{2$

# PART I: Visual Arts & Mathematics

## AGE RANGE: 16 – 18



Hypercube

(Source: Author at Desmos.com)

# TOOL 9: ART IMAGING THROUGH THE USE OF FUNCTIONS



SPEL – Sociedade Promotora de Estabelecimentos de Ensino



Co-funded by the Erasmus+ Programme of the European Union

## Educator's Guide

**Title:** Art imaging through the use of functions

**Age range:** 16 –18 years old

**Duration:** 2 hours

**Mathematical concepts:** Horizontal and vertical lines, absolute value, linear function, circle

**Artistic concepts:** Art imaging, graphic art

**General objectives:** To help students understand the concept of equations and be able to predict their graphical representation by experimenting on an online Cartesian plane.

**Instructions and Methodologies:** This tool will use the free, online graphic calculator Desmos, which will allow students to form shapes using a few basic functions and equations

**Resources:** Computer with an internet connection; Access to the website: <https://www.desmos.com/>

**Tips for the educator:** Have students see the graphical representation of horizontal and vertical lines, linear functions, absolute values and circumferences by explaining its concepts and showing examples on Desmos. Moreover, have students see how these graphical representations can expand or shrink, go left/right/downwards/upwards by changing their values and handing out the functions and equations for the alphabet letters, in order to guide them through the final task of this tool

**Learning Outcomes and Competences:** At the end of this lesson, the student will be able to:

- Graph equations and transformations of functions in the Cartesian plane;
- Identify the type of functions and, therefore, its potential outcome;
- Make graphical art through the use of functions

### Debriefing and Evaluation:

Write 3 aspects you liked about this activity:	1. 2. 3.
Write 2 aspects that you have learned	1. 2.
Write 1 aspect for improvement	1.

## Introduction

When people talk about making a graph, they are usually referring to the Cartesian graph. The Cartesian graph, also known as Cartesian coordinate system, is a plane used to pinpoint a coordinate on a plane through the use of two numbers: the x-coordinate and the y-coordinate.

When using equations on a Cartesian graph, one is able to draw images that make the relationship between visual arts and mathematics take on a higher level.

In this context, with the purpose of helping students understanding the concept of functions and their visualization simply by looking at them, the web application Desmos will be used as a canvas.

In this application, students will be able to play with graphical representation of equations by putting their creativity to work. Therefore, mathematics will be more personal and they will be able to relate to it.

## Art Imaging through the use of Functions

You have surely noticed from Math classes that graphical representations form shapes that sometimes resemble images and figures. Different functions produce different shapes, which means that in order to model images through graphs, it is necessary to understand the values in a function and the slopes and curves that result from it.

From simpler to complex graphical representations, by manipulating the length of such shapes and curves well enough, what is known as graphic art can be illustrated on a Cartesian coordinate system.

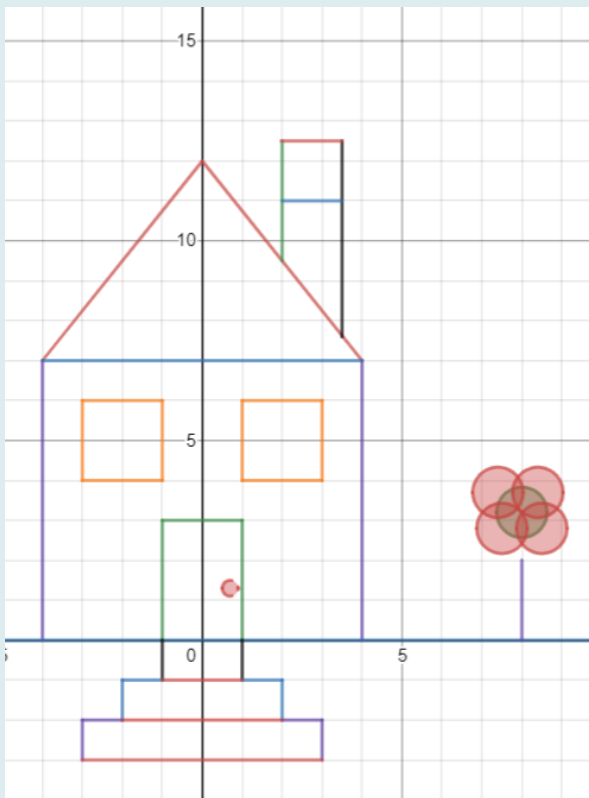


Fig. 1 – House and Flower

(Source: <https://www.desmos.com/calculator/9fahdexfkl>)

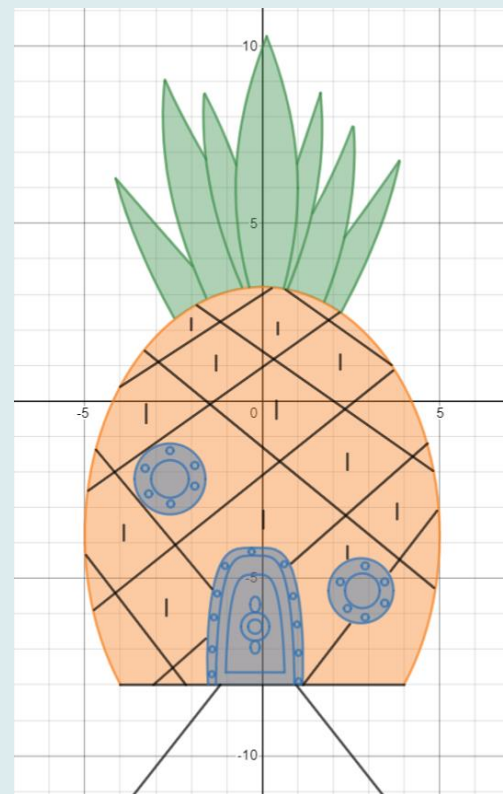


Fig. 2 – SpongeBob House (Source: Jordan Keckler; <https://www.desmos.com/calculator/z7d2cvdayo>)

Using the online graphic calculator Desmos as a canvas, this tool will focus on simple shapes, such as circles and straight horizontal, vertical and oblique lines, which is what it takes to produce graphical art as simple as the one in Fig. 1.

## Glossary

**Desmos:** an online scientific calculator software.

**Graphic art:** a type of visual artistic expression that focuses more on line and tone rather than on colour.

**Cartesian coordinate system:** a system used to specify any two points via numerical coordinates; often used for graphical representation; also referred as Cartesian graph.

# Maths behind Art Imaging

## 1. Horizontal and Vertical lines

As the name suggests, these are straight lines on a Cartesian coordinate plane. A horizontal line goes from left to right, always parallel to the x-axis, which means that all points in it have the same y-coordinate. On the other hand, a vertical line goes downward to upwards, always parallel to the y-axis and always with the same x-coordinate.

The equations that shape horizontal and vertical lines are:

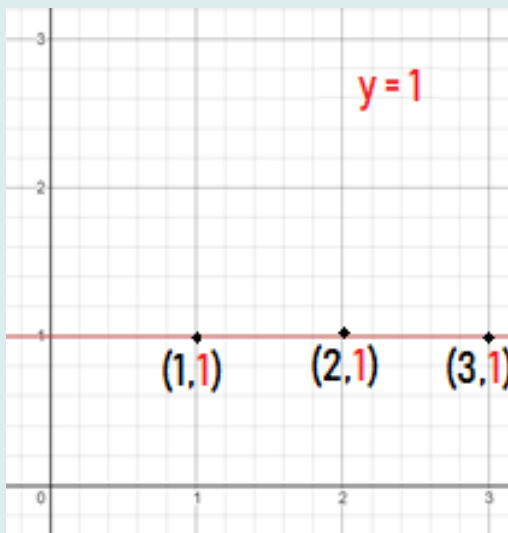
**Horizontal line**  
 $y = b$

**Vertical line**  
 $x = b$

**Where:**

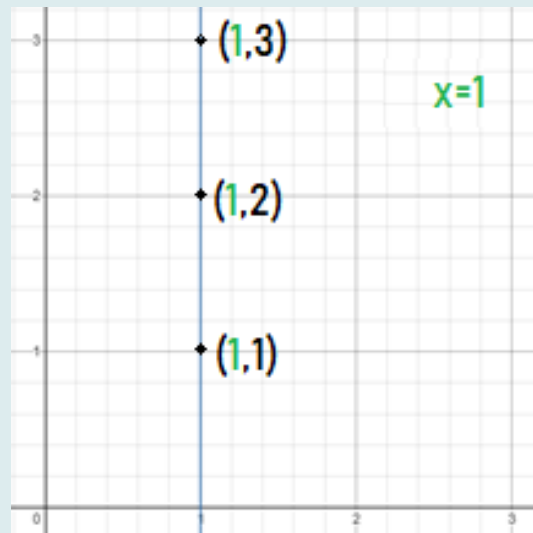
**b** is the point in which the line intercepts the y-axis (for the horizontal lines) or the x-axis (for the vertical lines).

**Horizontal line**



**Fig. 3 – Horizontal Line ( $y = 1$ )**  
(Source: Author, at Desmos.com)

**Vertical line**



**Fig. 4 – Vertical Line ( $x = 1$ )**  
(Source: Author, at Desmos.com)

As seen, in horizontal lines, no matter what the x value is going to be, the y-intercept is always 1. Similarly, it is not important what the y value is on vertical lines, the x-intercept is always 1.

## 2. Linear Functions

Just like horizontal and vertical lines, linear functions are those whose graphical representation form a straight line in a plane. However, they are not always constant. They can also have an increasing or a decreasing shape.

They are formed by the following equation:

$$y = mx + b$$

### Where:

**m** is a constant and represents the slope (or gradient) of a line;

**b**, known as y-intercept, determines the point at which the line crosses the y-axis.

### When:

**m > 0** the slope of the line will represent an increase;

**m = 0** the slope of the line is constant (becoming a horizontal line);

**m < 0** the slope of the line will represent a decrease.

Linear functions can be used to depict and predict many applications and situations in life in which there is an unknown value/quantity.

**Example:** you are planning to have your birthday in a party saloon. Even though you know the service costs €9 and the meal cost per person is €8.15, you are still unsure of how many of your friends are coming.

In this case, your linear function will be

$$y = 8.15x + 9$$

**Slope:  $m = 8.15$**

**Intercept:  $b = 9$**

As of this moment, when replacing  $x$  for the number of participants, the result will be the total cost. For instance, if 4 people are coming (i.e., when  $x = 4$ ),  $y$  (i.e. the total cost) is €25.60.

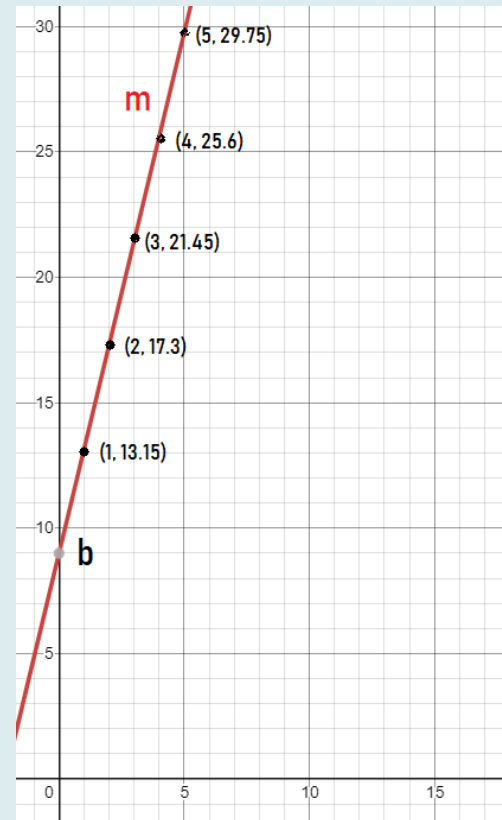


Fig. 5 – Linear Function ( $y = 8.15x + 9$ )  
(Source: Author, at Desmos.com)

### 3. Absolute value

The absolute value of a number can be thought as its distance to its origin 0, which, therefore, means that any number, weather negative or positive, will always have a positive value of itself.

An absolute value of a number is represented in the following way:

$$|x|$$

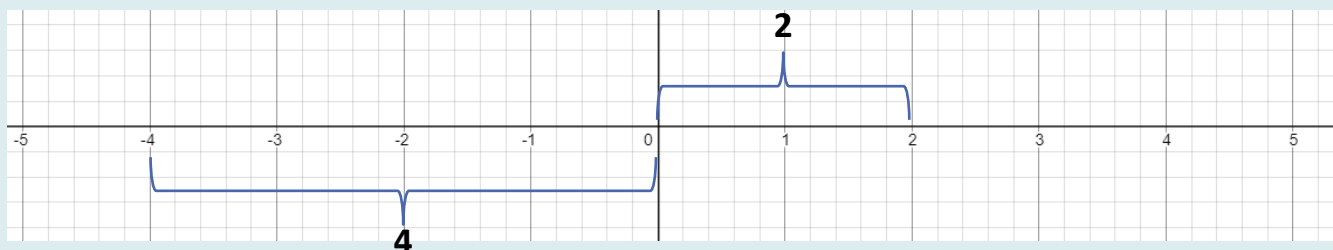


Fig. 6 – Absolute value graphical representation  
(Source: Author, at Desmos.com)



This way, instead of writing “the absolute value of -4 is 4”, we can simply write “ $|-4| = 4$ ”.

This graph shows the absolute value function for real numbers.

Essentially, it states that  $|x|$  corresponds to its y-axis number, that is, for instance,  $|-2|$  (the absolute value of -2) is 2.

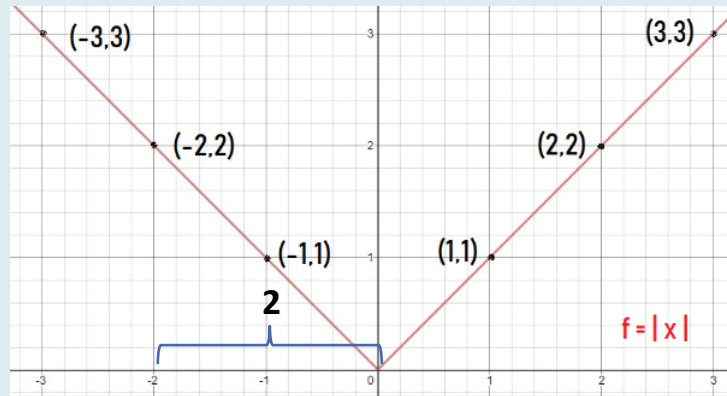


Fig. 7 – Absolute value ( $f = |x|$ )  
(Source: Author, at Desmos.com)

## 4. Circles

When you know a circle's radius and the coordinates of its center, it is possible to graph a circle in a Cartesian coordinate system.

The equation of a circle is:

$$(x - x_c)^2 + (y - y_c)^2 = r^2$$

**Where:**

$x$  and  $y$  are any points on the circle;

$x_c$  and  $y_c$  are the points in the center of the circle, corresponding to each axis;

$r$  is the radius.

Suppose you want to graph a circle with the radius of 3, given the coordinates of the center is 0 for both its x and y axes.

$$(x - 0)^2 + (y - 0)^2 = 3^2$$

In the given example, the result would be the following graph:

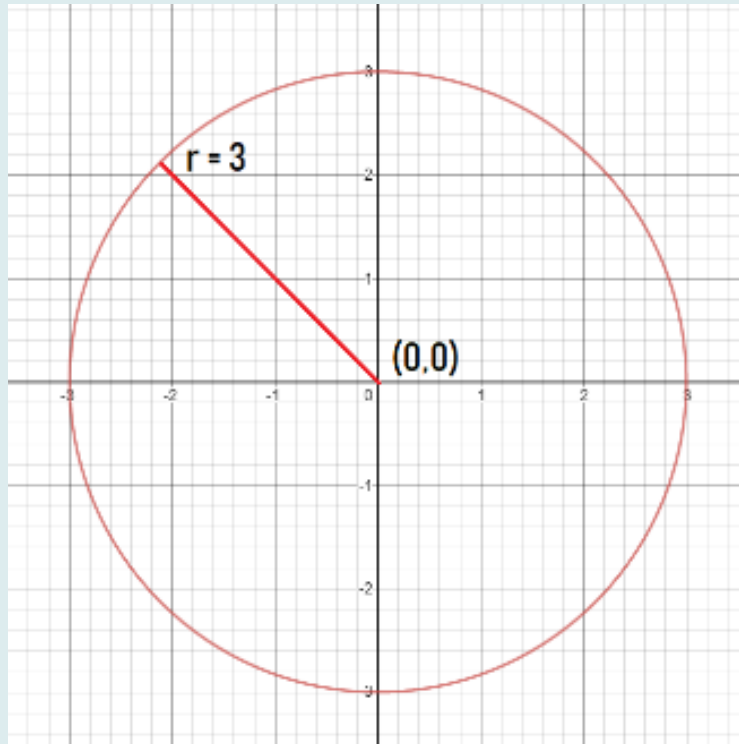
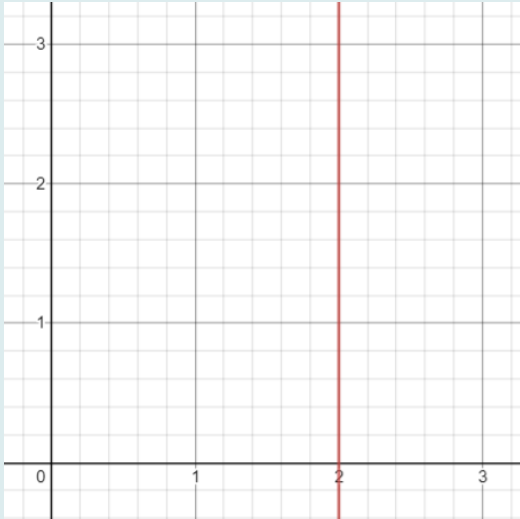


Fig. 8 – Circle  $(x - 0)^2 + (y - 0)^2 = 3^2$   
(Source: Author, at Desmos.com)

## Working with Desmos

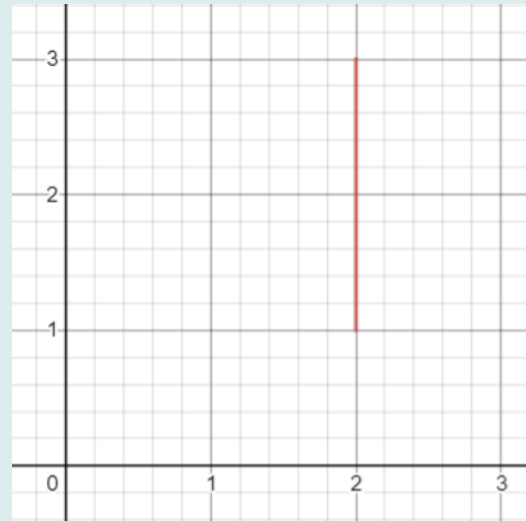
When working with Desmos, if you want to omit part of a graph you will need to set limits to the function – this is done by setting your values between brackets. Consider the following examples:

$x = 2$  forms a vertical line that will go from  $-\infty$  to  $+\infty$  while crossing 2 in the x-coordinate.

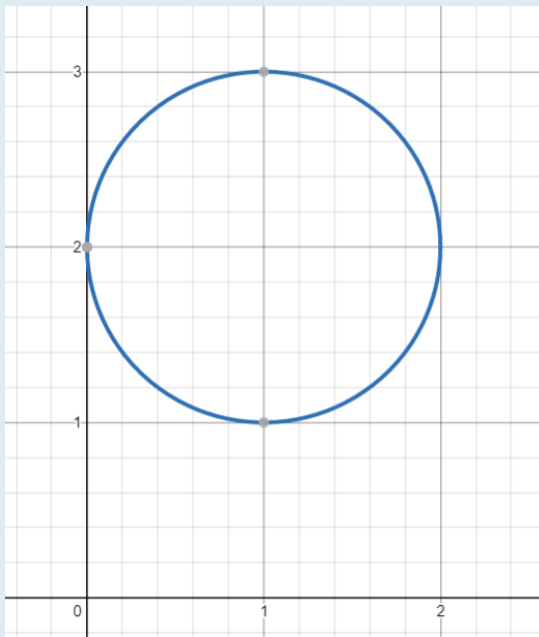


$x = 2 \{ 1 < y < 3 \}$

However, in this case, the vertical line was set to show only where  $y > 1$  and  $y < 3$ .

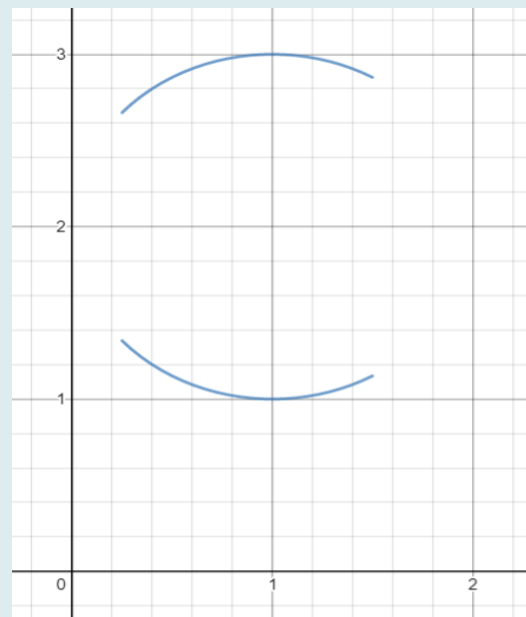


$(x-1)^2 + (y-2)^2 = 1^2$  forms the shape of a circle where the center of itself was set as  $x = 1$  and  $y = 2$ , with a radius of 1.



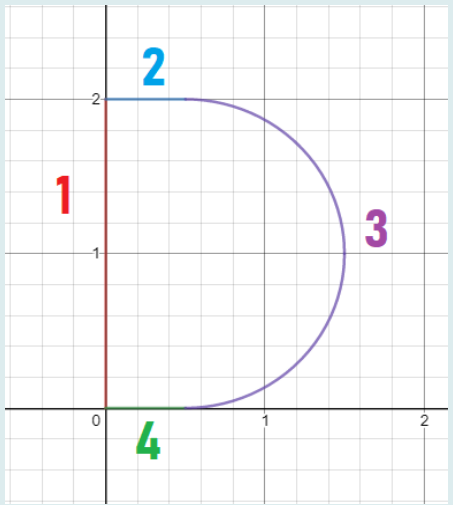
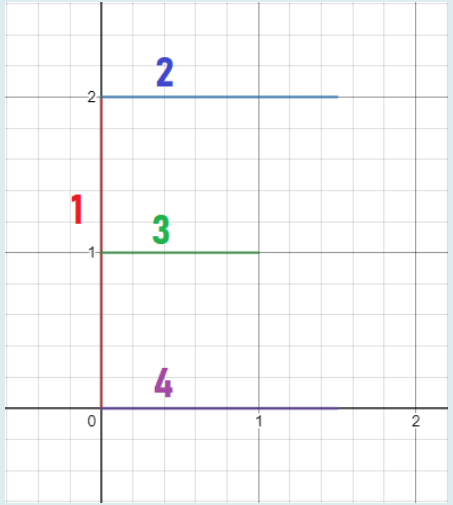
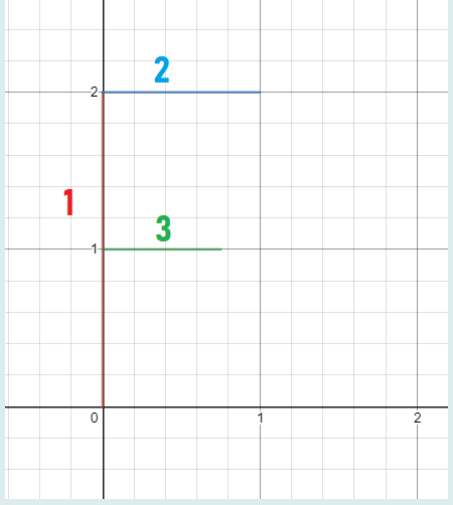
$(x-1)^2 + (y-2)^2 = 1^2 \{ 0.25 < x < 1.5 \}$

Using the same circle equation, but setting it to show only where  $x > 0.25$  and  $x < 1.5$ , results in the following visual:



Analyse the functions used to create the alphabet and then proceed to the final task.

<p>1) <math>2x + 0 \{0 &lt; x &lt; 1\}</math></p> <p>2) <math>-2x + 4 \{1 &lt; x &lt; 2\}</math></p> <p>3) <math>y = 1 \{0.5 &lt; x &lt; 1.5\}</math></p>	<h1>A</h1>	
<p>1) <math>x = 0 \{0 &lt; y &lt; 2\}</math></p> <p>2) <math>y = 2 \{0 &lt; x &lt; 0.5\}</math></p> <p>3) <math>(x - 0.5)^2 + (y - 1.5)^2 = 0.25 \{0.5 &lt; x &lt; 1\}</math></p> <p>4) <math>y = 1 \{0 &lt; x &lt; 0.5\}</math></p> <p>5) <math>(x - 0.5)^2 + (y - .5)^2 = 0.25 \{0.5 &lt; x &lt; 1\}</math></p> <p>6) <math>y = 0 \{0 &lt; x &lt; 0.5\}</math></p>	<h1>B</h1>	
<p>1) <math>(x - 1)^2 + (y - 1)^2 = 1 \{x &lt; 1.75\}</math></p>	<h1>C</h1>	

<p>1) <math>x = 0 \{0 &lt; y &lt; 2\}</math></p> <p>2) <math>y = 2 \{0 &lt; x &lt; 0.5\}</math></p> <p>3) <math>y = 0 \{0 &lt; x &lt; 0.5\}</math></p> <p>4) <math>(x - 0.5)^2 + (y - 1)^2 = 1 \{x &gt; 0.5\}</math></p>	<h1>D</h1>	
<p>1) <math>x = 0 \{0 &lt; y &lt; 2\}</math></p> <p>2) <math>y = 2 \{0 &lt; x &lt; 1.5\}</math></p> <p>3) <math>y = 1 \{0 &lt; x &lt; 1\}</math></p> <p>4) <math>y = 0 \{0 &lt; x &lt; 1.5\}</math></p>	<h1>E</h1>	
<p>1) <math>x = 0 \{0 &lt; y &lt; 2\}</math></p> <p>2) <math>y = 2 \{0 &lt; x &lt; 1\}</math></p> <p>3) <math>y = 1 \{0 &lt; x &lt; 0.75\}</math></p>	<h1>F</h1>	

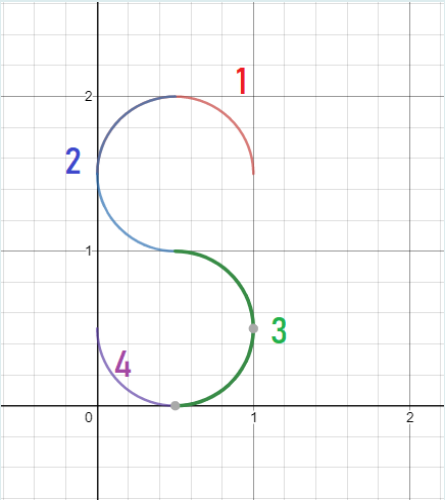
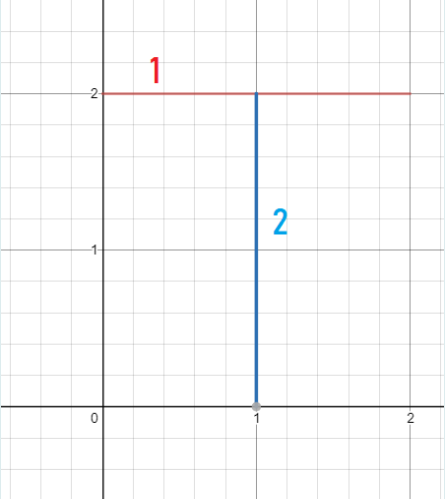
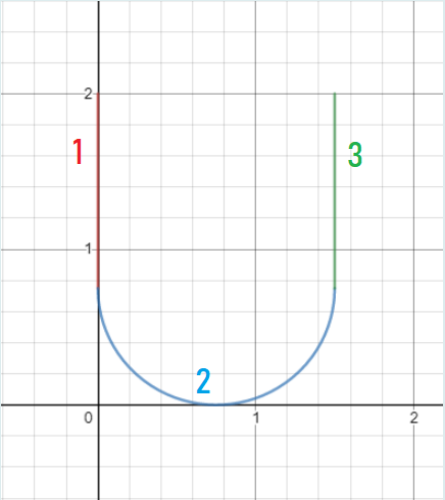
<p>1) <math>(x - 1)^2 + (y - 1)^2 = 1 \{x &lt; 1.5\}</math></p> <p>2) <math>y = 1 \{1 &lt; x &lt; 1.5\}</math></p> <p>3) <math>x = 1.5 \{0.134 &lt; y &lt; 1\}</math></p>	<h1>G</h1>	
<p>1) <math>x = 0 \{0 &lt; y &lt; 2\}</math></p> <p>2) <math>y = 1 \{0 &lt; x &lt; 1\}</math></p> <p>3) <math>x = 1 \{0 &lt; y &lt; 2\}</math></p>	<h1>H</h1>	
<p>1) <math>y = 2 \{0 &lt; x &lt; 1\}</math></p> <p>2) <math>x = 0.5 \{0 &lt; y &lt; 2\}</math></p> <p>3) <math>y = 0 \{0 &lt; x &lt; 1\}</math></p>	<h1>I</h1>	

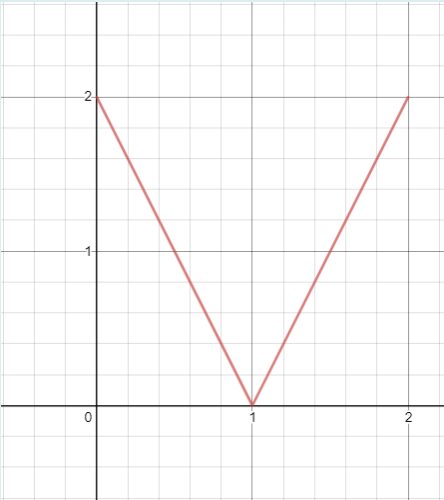
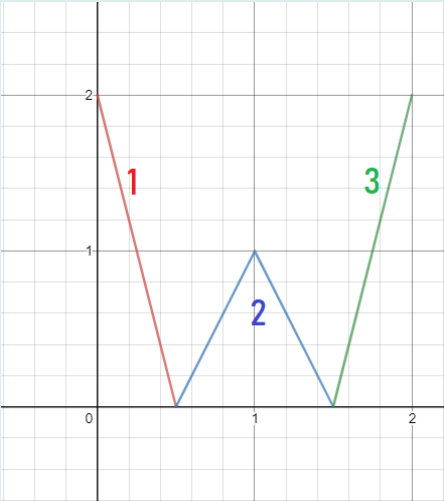
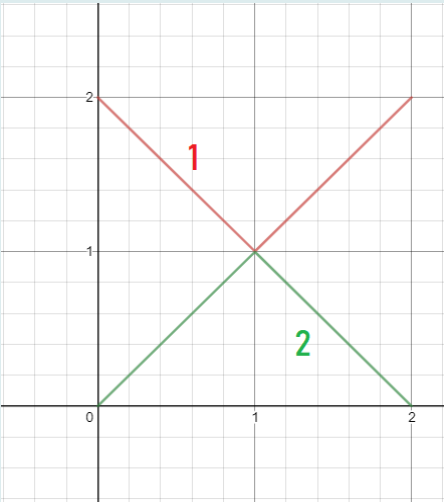
<p>1) <math>y = 2 \{0.5 &lt; x &lt; 1.5\}</math></p> <p>2) <math>x = 1 \{0.5 &lt; y &lt; 2\}</math></p> <p>3) <math>(x - 0.5)^2 + (y - 0.5)^2 = 0.25 \{0 &lt; y &lt; 0.5\}</math></p>	<h1>J</h1>	
<p>1) <math>x = 0 \{0 &lt; y &lt; 2\}</math></p> <p>2) <math>x + 1 \{0 &lt; x &lt; 1\}</math></p>	<h1>K</h1>	
<p>1) <math>x = 0 \{0 &lt; y &lt; 2\}</math></p> <p>2) <math>y = 0 \{0 &lt; x &lt; 1\}</math></p>	<h1>L</h1>	

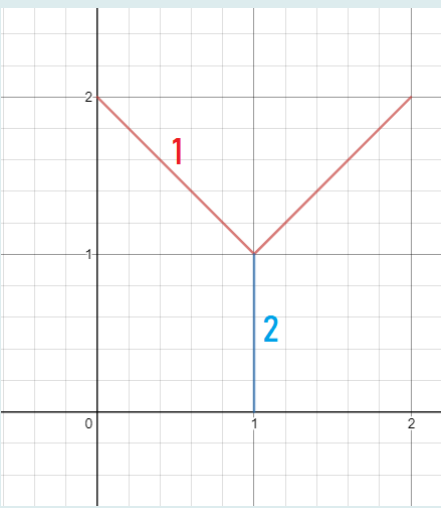
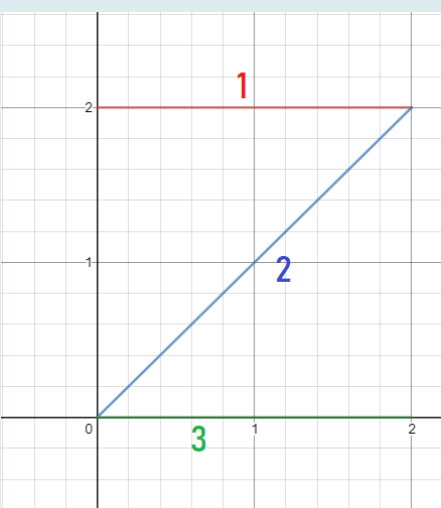
<p>1) <math>x = 0 \{0 &lt; y &lt; 2\}</math></p> <p>2) <math>-x + 2 \{0 &lt; x &lt; 1\}</math></p> <p>3) <math>x \{1 &lt; x &lt; 2\}</math></p> <p>4) <math>x = 2 \{0 &lt; y &lt; 2\}</math></p>	<h1>M</h1>	
<p>1) <math>x = 0 \{0 &lt; y &lt; 2\}</math></p> <p>2) <math>-4/3x + 2 \{0 &lt; x &lt; 1.5\}</math></p> <p>3) <math>x = 1.5 \{0 &lt; y &lt; 2\}</math></p>	<h1>N</h1>	
<p>1) <math>(x - 1)^2 + (y - 1)^2 = 1</math></p>	<h1>O</h1>	



<p>1) <math>x = 0 \{0 &lt; y &lt; 2\}</math></p> <p>2) <math>y = 2 \{0 &lt; x &lt; 1\}</math></p> <p>3) <math>(x - 1)^2 + (y - 1.5)^2 = 0.25 \{1 &lt; x &lt; 1.5\}</math></p> <p>4) <math>y = 1 \{0 &lt; x &lt; 1\}</math></p>	<h1>P</h1>	
<p>1) <math>(x - 1)^2 + (y - 1)^2 = 1</math></p> <p>2) <math>-x + 2 \{1.5 &lt; x &lt; 2\}</math></p>	<h1>Q</h1>	
<p>1) <math>x = 0 \{0 &lt; y &lt; 2\}</math></p> <p>2) <math>y = 2 \{0 &lt; x &lt; 0.5\}</math></p> <p>3) <math>(x - 0.5)^2 + (y - 1.5)^2 = 0.25 \{x &gt; 0.5\}</math></p> <p>4) <math>y = 1 \{0 &lt; x &lt; 0.5\}</math></p> <p>5) <math>-2x + 2 \{0.5 &lt; x &lt; 1\}</math></p>	<h1>R</h1>	

<p>1) <math>(x - 0.5)^2 + (y - 1.5)^2 = 0.25</math>  <math>\{y &gt; 1.5\}</math></p> <p>2) <math>(x - .5)^2 + (y - 1.5)^2 = .25 \{x &lt; .5\}</math></p> <p>3) <math>(x - 0.5)^2 + (y - 0.5)^2 = .25 \{0.5 &lt; x &lt; 1\}</math></p> <p>4) <math>(x - 0.5)^2 + (y - 0.5)^2 = .25 \{y &lt; 0.5\}</math></p>	<h1>S</h1>	
<p>1) <math>y = 2 \{0 &lt; x &lt; 2\}</math></p> <p>2) <math>x = 1 \{0 &lt; y &lt; 2\}</math></p>	<h1>T</h1>	
<p>1) <math>x=0 \{0.75 &lt; y &lt; 2\}</math></p> <p>2) <math>(x - 0.75)^2 + (y - 0.75)^2 = 0.75^2 \{y &lt; 0.75\}</math></p> <p>3) <math>x=1.5 \{0.75 &lt; y &lt; 2\}</math></p>	<h1>U</h1>	

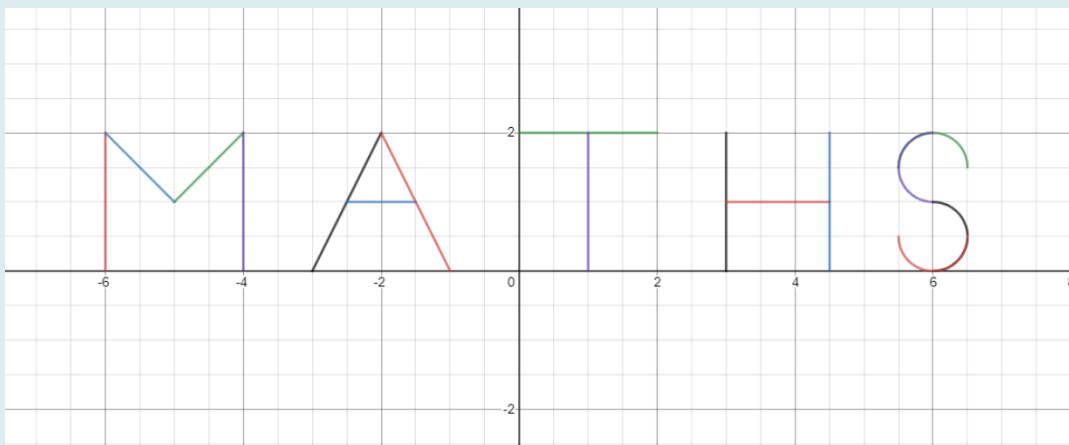
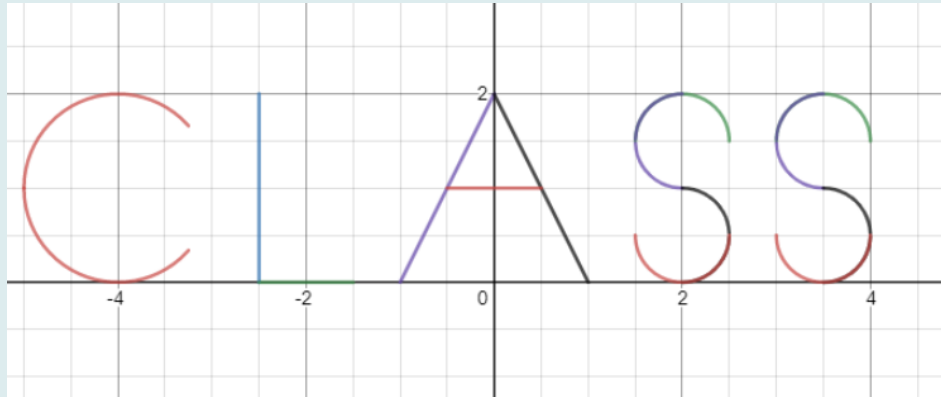
<p>1) <math>2 x - 1  \{0 &lt; x &lt; 2\}</math></p>	<p>V</p>	
<p>1) <math>-4x + 2 \{0 &lt; x &lt; 0.5\}</math>                  2) <math>-2 x - 1  + 1 \{0.5 &lt; x &lt; 1.5\}</math>                  3) <math>4x - 6 \{1.5 &lt; x &lt; 2\}</math></p>	<p>W</p>	
<p>1) <math> x - 1  + 1 \{0 &lt; x &lt; 2\}</math>                  2) <math>-x - 1  + 1 \{0 &lt; x &lt; 2\}</math></p>	<p>X</p>	

<p>1) <math> x - 1  + 1 \{0 &lt; x &lt; 2\}</math></p> <p>2) <math>x = 1 \{0 &lt; y &lt; 1\}</math></p>	<h1>Y</h1>	
<p>1) <math>y = 2 \{0 &lt; x &lt; 2\}</math></p> <p>2) <math>x \{0 &lt; x &lt; 2\}</math></p> <p>3) <math>y = 0 \{0 &lt; x &lt; 2\}</math></p>	<h1>Z</h1>	

# TASKS

## TASK 1

By using the equations given to form the letters of the alphabet, produce the words "CLASS" and "MATHS".



## TASK 2

If you were successful with question 1, put your creativity to work and graph an image to your taste or write your own name.

## LEARN MORE...

Explore math with Desmos web application

<https://www.desmos.com/>

The alphabet written with equations in Desmos

<https://www.desmos.com/calculator/l8u2vigxyb>

Graphing absolute value functions

<https://www.khanacademy.org/math/algebra/absolute-value-equations-functions/graphs-of-absolute-value-functions/v/graphing-absolute-value-functions>

Derivatives and Integrals in Art Imaging

[https://ethnomath.coe.hawaii.edu/pdf/university\\_derivatives\\_integrals.pdf](https://ethnomath.coe.hawaii.edu/pdf/university_derivatives_integrals.pdf)

Introduction to Linear Functions

<https://courses.lumenlearning.com/boundless-algebra/chapter/introduction-to-linear-functions/>

The Geometry of a Circle

<http://www.mathcentre.ac.uk/resources/uploaded/mc-ty-circles-2009-1.pdf>