TOOL 55: UNCLE PETROS AND GOLDBACH'S CONJECTURE

C.I.P. Citizens in Power

TheArtOf
Maths

Educator's Guide

Title: Uncle Petros and Goldbach's Conjecture

Age Range: 13-15 years old

Duration: 1.5 hour

Mathematical Concepts: Number Theory/ Even numbers/ Prime Numbers

Artistic Concepts: Novel

General Objectives: This tool is attempting to aspire students. If they get intrigued to read the book they will see through the eyes of a young fellow his role model, a great mathematician.

Instructions and Methodologies: It is preferable to follow the structure of this tool as it begins with some simple background information on the connection between literacy and mathematics in general, whilst getting into more details and its benefits progressively. A biography of the author and an overview of the book are given before reaching some excerpts from the book. Then a simple example is given based on the conjecture, a 'let's try section where students can use the pyramid and then the more demanding Mathematical tasks.

Resources: This tool provides pictures and sites as resources as well as a glossary.

Tips for the educator: Try to encourage the students to read the whole book and come back to you with more insights, encourage discussion or even a presentation. Students should be familiar with what prime numbers are before undertaking any of the tasks presented and therefore understanding the logic behind Goldbach's Conjecture.

Desirable Outcomes and Competences: At the end of this tool, the student will be able to:

- Know what it is meant by Goldbach's Conjecture
- Exercise on prime numbers through meaningful tasks

Debriefing and Evaluation:

Write 3 aspects you liked about this activity:	1. 2. 3.
Write 2 aspects that you have learned	1. 2.
Write 1 aspect for improvement	1.

Introduction

According to Cohen (2013) "Studying mathematics-related fiction and poetry helps students develop an appreciation for both mathematics and literature and an understanding of the connection between the two". There are many studies coming from researchers such as from Growney (2008, 2009), Bahls (2009), Glaz and Liang (2009), Glaz (2010, 2011), and Ivy (2004, 2009) explaining how mathematics can be combined with literature in the classroom. Generally connecting mathematics with arts, such as cinematography, drama, and language arts, has been considered as a helpful strategy for instructing mathematics for several reasons. Research has proven that providing an environment that is considered to be less stressful and psychologically safe increases students' inspiration and results (Jensen, 1998). When students are worried, achievement lessens because they are preoccupied with a nervous and worrying feeling, distracting them from the actual mathematical tasks (Covington 1999). These students could miss a lot of the information they are intended to learn because their focus is derived and deprived from their fear of mathematics, instead of the task to be learnt (Siegel 1999). Using literature effectively with mathematics can help reduce the anxiety felt by 'mathephobes' (Zambo, 2005).
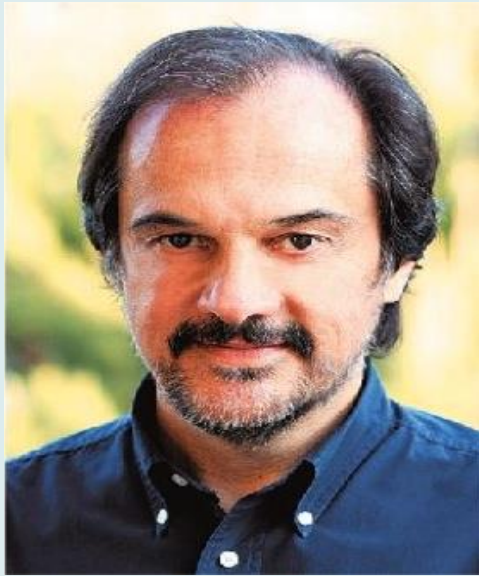
There are many literacy books in several languages associated with mathematics. Giving a small number of the most known ones are: 'The Devotion of suspect X' by Keigo Higashino (1958), 'The calculus wars' by Jason Socrates Bardi (2006),

'Logicomix' also by Apostolos Doxiades (2009), 'Mathematical Mysteries: The Beauty and Magic of Numbers' by Calvin Clawson (1999) , 'The housekeeper and the Professor' by Yoko Ogawa (2009) a bestselling Japanese novel also translated in English and the one chosen for this tool 'Uncle Petros and Goldbach's conjecture'.

The novel Uncle Petros and the Goldbach's Conjecture by Apostolos Doxiades Greek version (1992), English version (2000), is giving mathematical problems and some recent history of mathematics. It is a story about a great mathematician and it is not intending to teach mathematics per se. It is not a great sample of literature -as this is the case with most novels-. Albeit that, it has drama and suspense. It will be great for the students in this particular age range to read this book. This may aspire them because of the great personality of Petros as a mathematician; the example he formed and the impact he had on his nephew. It is not the main purpose for students to learn mathematical concepts through this book neither to deepen their knowledge on prime numbers.

Biography of the writer

Doxiadis was born in Australia, where his father, the architect Constantinos Apostolou Doxiadis was working. Soon after his birth, the family returned to Athens, where Doxiadis grew up. Though his earliest interests were in poetry, fiction and the theatre, an intense interest in mathematics led Doxiadis to leave school at age fifteen, to attend Columbia University, in New York, from which he obtained a bachelor's degree in Mathematics in May 1972. He then attended the École Pratique des Hautes Études in Paris from which he got a master's degree[citation needed], with a thesis on the mathematical modeling of the nervous system. His father's death and family reasons made him return to Greece in 1975, interrupting his graduate studies. In Greece, although involved for some years with the computer software industry, Doxiadis returned to his childhood and adolescence loves of theatre and the cinema, before becoming a full-time writer.

Uncle Petros and Goldbach's Conjecture was the first recipient of the Premio Peano the first international award for books inspired by mathematics and short-listed for the Prix Médicis. Logicomix has earned numerous awards, among them the Bertrand Russell Society Award, the Royal Booksellers Association Award (the Netherlands), the New Atlantic Booksellers Award (USA), the Prix Tangente (France), the Premio Carlo Boscarato (Italy), the Comicdom Award (Greece). It was chosen as "Book of the Year" by TIME Magazine, Publishers Weekly, The Washington Post, The Financial Times, The Globe and Mail, and other publications.

Original Source from:

https://en.wikipedia.org/wiki/Apostolos_Doxiadis

Overview of the book 'Uncle Petros and Goldbach's Conjecture'

The book gives suspense and drama through the eyes of a kid whose uncle is a gifted Greek mathematician named Petros Papachristos, who devoted his life in proofing Goldbach's Conjecture. Uncle Petros is a fictional character but, a number of real-life mathematicians appear as characters in the book, including Constantin Carathéodory, G. H. Hardy, J. E. Littlewood, Srinivasa Ramanujan, and Kurt Gödel. From when he was young Uncle Petros is obsessed with solving Goldbach's Conjecture and devotes all his efforts in life towards this direction.

> *'This single-minded devotion to a wild dream is what makes Uncle Petros an appealing and tragic figure. It is also what makes him a mathematician' Mathematical Association of America, Volume 47, No.10, p.1275*

Uncle Petros is a pure mystery. The elders of the Papachristou family reject him as a "failure of life". Until his narrator-nephew discovers that he was once a famous mathematician, so genius and daring as to dedicate his life to the infamous "Goldbach's conjecture", a problem that several generations of mathematicians have been trying in vain to solve. This discovery will lead to a series of reactions. As Petros recounts his own life's work, a bond is formed between uncle and nephew, dragging each one deeper into mathematical obsession, and endangering uncle's and nephew's sanity.

Excerpts from 'Uncle Petros and Goldbach's Conjecture'

> Although Uncle Petros remained expressionless, I noticed a slight tremor run down his hand.
>
> "Who's spoken to you about Goldbach's Conjecture?" he asked quietly.
>
> "My father," I murmured.
>
> And what did he say, precisely?"
>
> "That you tried to prove it."
>
> "Just that?"
>
> "And.... that you didn't succeed."
>
> His hand was steady again. "Nothing else?"
>
> "Nothing else."
>
> "Hm," he said. "Suppose we make a deal?"
>
> "What sort of deal?"

Retrieved from: https://www.maa.org/press/maa-reviews/uncle-petros-and-goldbachs-conjecture

Glossary

Goldbach's conjecture is one of the oldest and best-known unsolved problems in number theory and all of mathematics. It states:

Every even integer greater than 2 can be expressed as the sum of two primes

The conjecture has been shown to hold for all integers less than 4×10^{18} but remains unproven despite considerable effort.

Retrieved from: https://en.wikipedia.org/wiki/Goldbach%27s_conjecture

Binary: the numerical notation system that uses 2 as a base.

The Math behind Uncle Petros and Goldbach's Conjecture

Even number: any whole number that can be exactly divided by 2.
Example: 2, 4, 6, 8

Prime number: a whole number greater than 1 that is divisible only by itself and 1.
Example: 2, 3, 5, 7, 11

The Goldbach Conjecture is still not proven, stating basically that every even integer greater than two is the sum of two prime numbers. The conjecture has been tested up to 400,000,000,000,000.

Note that, many even numbers have one or more possible ways of writing them as the sum of two primes.

For example:

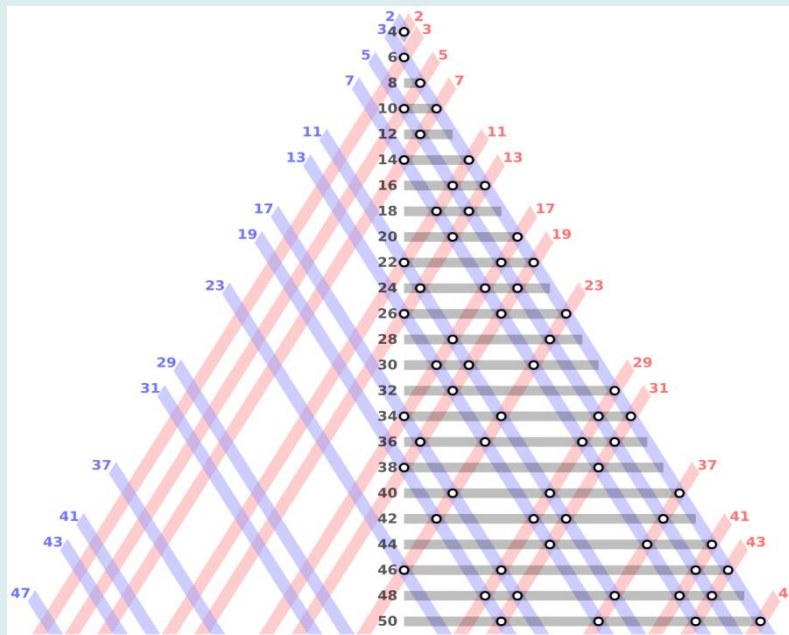$$4 = 2 + 2$$
$$6 = 3 + 3$$
$$8 = 3 + 5$$
$$10 = 3 + 7 = 5 + 5$$
$$12 = 5 + 7$$
$$14 = 3 + 11 = 7 + 7$$
...

Goldbach's conjecture is one of the oldest unsolved problems in number theory and in all of mathematics.

Picture 2: Goldbach's Conjecture. The ends represent the prime numbers meeting at their respective sums
(Retrieved from: https://towardsdatascience.com/prime-numbers-and-goldbach-s-conjecture-visualization-60d1993a1424

Origins

In 1742, the Prussian mathematician Christian Goldbach wrote a letter to Leonhard Euler in which he proposed the following conjecture:

Every integer greater than 2 can be written as the sum of three primes.

He considered 1 to be a prime number, a convention subsequently abandoned. So today, Goldbach's original conjecture would be written:

Every integer greater than 5 can be written as the sum of three primes.

Euler, becoming interested in the problem, reminded Goldbach of an earlier conversation they had in which Goldbach remarked his original conjecture:

Every even number greater than 2 can be written as the sum of two primes, adding that he regarded this a fully certain theorem ("ein ganz gewisses Theorema"), in spite of his being unable to prove it.

The former conjecture is today known as the "ternary" Goldbach conjecture, the latter as the "strong" or "binary" Goldbach conjecture. The conjecture that all odd integers greater than 9 are the sum of three odd primes is called the "weak" Goldbach conjecture.

Both questions have remained unsolved ever since, although the weak form of the conjecture is much closer to resolution than the strong one.

The majority of mathematicians believe the conjecture (in both the weak and strong forms) to be true, at least for sufficiently large integers, mostly based on statistical considerations focusing on the probabilistic distribution of prime numbers: the bigger the number, the more ways there are available for that number to be represented as the sum of two or three other numbers, and the more "likely" it becomes that at least one of these representations consists entirely of primes.

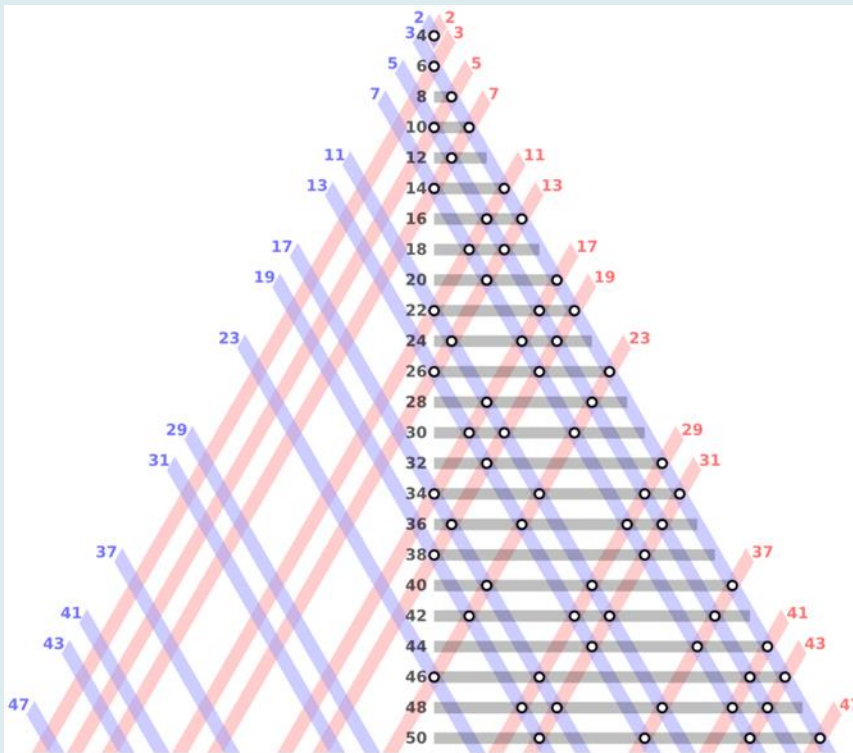Retrieved from: https://artofproblemsolving.com/wiki/index.php/Goldbach_Conjecture

Let's try!

Find the two prime numbers that add up to the following even integer:

    a) 46 =

    b) 38 =

    c) 14 =

    d) 22 =

    e) 40 =

(Hint: you can use the following pyramid)

Description: The numbers at the center of the pyramid represent the even numbers from 2 - 50. Following the grey line on the right-hand side of the pyramid helps you to find the two prime number (purple and red) that add up to the even number.

TASKS

TASK 1

The following even integers are broken into two numbers. **Are the two numbers prime? Tick the box. If NO please provide the correct answer.**

a) $52 = 23 + 29$

YES ☐ NO ☐ Correct Answer:

b) $76 = 9 + 67$

YES ☐ NO ☐ Correct Answer:

c) $80 = 59 + 21$

YES ☐ NO ☐ Correct Answer:

d) $120 = 73 + 47$

YES ☐ NO ☐ Correct Answer:

e) $64 = 19 + 45$

YES ☐ NO ☐ Correct Answer:

f) $92 = 89 + 3$

YES ☐ NO ☐ Correct Answer:

TASK 2

The following even integers are broken down into two prime numbers. **Give a different answer from the one given.**

a) $90 = 31 + 59 =$

b) $56 = 3 + 53 =$

c) $88 = 71 + 17 =$

d) $202 = 11 + 191 =$

e) $62 = 19 + 43 =$

f) $94 = 11 + 83 =$

g) $110 = 3 + 107 =$

LEARN MORE...

Book review by the Mathematical Association of America

https://www.ams.org/notices/200010/rev-jackson.pdf

Goldbach's Conjecture Explained:

https://artofproblemsolving.com/wiki/index.php/Goldbach_Conjecture