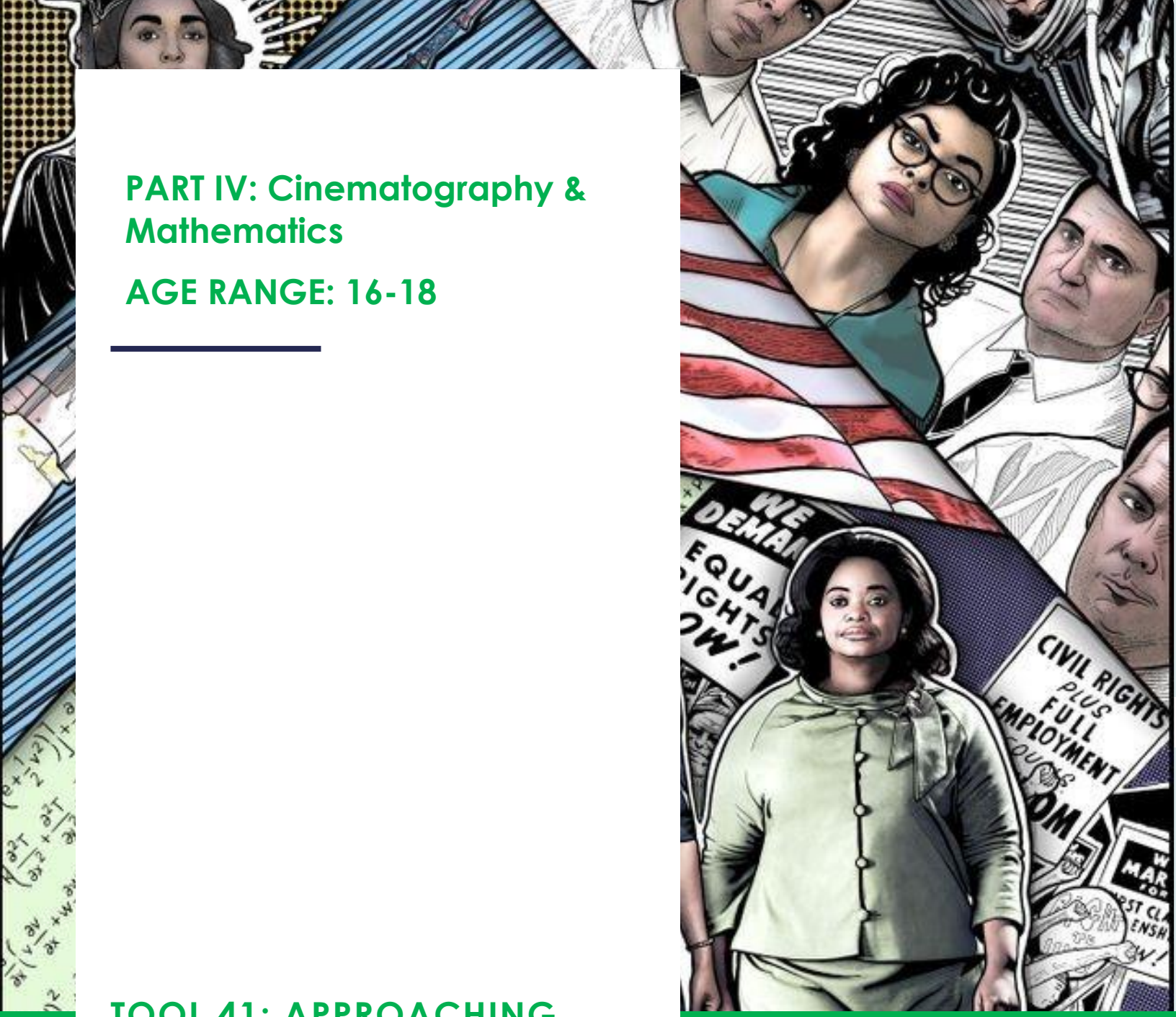

PART IV: Cinematography & Mathematics

AGE RANGE: 16-18

TOOL 41: APPROACHING DERIVATIVE OF A FUNCTION THROUGH THE MOVIE "HIDDEN FIGURES"

C.I.P. Citizens In Power

TheArtOf Maths

Co-funded by the Erasmus+ Programme of the European Union

Educator's Guide

Title: Approaching Derivative of a function through the movie '*Hidden Figures*'

Age Range: 16-18 years old

Duration: 1 hour

Mathematical Concepts: Euler's Method, Definition of Derivative

Artistic Concepts: Cinematography.

General Objectives: The students, through a non-stressful environment, will familiarize with concepts both inside and outside the mathematical area. Even through the 3-minute trailer, students will get aspired of the huge contribution of colored women in the USA, in the beginning of the 20th century, along with their route to make history. Ultimately the students will also practice some mathematical concepts from algebra, namely the definition of derivative.

Instructions and Methodologies: It is preferable to follow the structure of this tool, as it begins with some simple background information on the connection between mathematics and cinematography in general, whilst getting into more details on the specific movie used. Firstly, it would be nice to see the pictures of the three female protagonists and read about their biography from the glossary. Then the plot is given; it can be read individually by each student or aloud in the class, before having the chance to see the actual trailer of the movie, which may lead to a brainstorming activity. The educator can raise both social issues (also briefly mentioned in the introduction) and mathematical issues; e.g asking the students what they believe to be the role of mathematics in rocket science, before reading the information on the 'Maths Behind' section. Subsequently, getting towards the end, there are two tasks: the 1st one can be done as homework, asking students to learn more about a significant mathematician called Leonhard Euler; and Task 2 which is an actual mathematical exercise based on the concept of derivative.

Resources: Biography of the three female protagonists, the trailer of the movie, some pictures and two tasks.

Tips for the educator: It is important to show the students the trailer from the beginning in order to capture their attention. It is also crucial to emphasize both the

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sociological stance of the movie (presenting the persuasiveness, courage, persistency, devotion of the '*female computers*' as they have been called, by combining hard work and having a family), but also other issues mentioned earlier, such as the obstacles of being a woman and an African-American at that time in the USA.

Desirable Outcomes and Competences:

- To acknowledge that mathematics -from their simplest form to their more complicated is important, not only for our everyday lives but also responsible for the greatest accomplishments of human history, made known through this movie, such as the launch into the Orbit and its safe return.
- Use the formula that estimates the derivative of a function

Debriefing and Evaluation Questions:

You can use these cards sometimes called EXIT CARDS either by a hard copy you have made from before or simply by posing these statements on board and the students write the answers on a paper which they will leave preferably anonymously while exiting the room. The specific formative strategy is called 3,2,1. For more strategies you can visit:

https://www.bhamcityschools.org/cms/lib/AL01001646/Centricity/Domain/131/70%20Formative%20Assessments.pdf

3-2-1	
Write 3 things you liked about this activity	**1.** **2.** **3.**
Write 2 things you have learned	**1.** **2.**
Write 1 aspect for improvement	**1.**

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Introduction

According to Polster (2012) there are more than 700 movies related to mathematics to a big or smaller extent. Movies are considered as an injection of moments of fun, which can be used into courses in an attempt to make the learning of mathematics amusing and interesting for the young audiences. For this task, a recent movie (2016) called the *'Hidden Figures'* has been chosen for several reasons.

One of the reasons is due to its social messages of empowerment of women and people of color in the beginning of the 20th century. Lots of discussions according to several aspects such as if time permits, the composition of the classroom and the age of the students can arise from this film (even from just viewing the trailer of the movie; through the link given later on this tool). Some of those sociological issues are, the race, the gender, combining family and career, the differences (if any) between the now and then, between countries etc.

Secondly, it can elaborate discussions and small projects/ research and/or presentation on a famous mathematician called Leonhard Euler, on whose methods (used in another field originally) were useful and used in the field of rocket science, shown in this movie.

Thirdly, as stated, by Kirk Long (2017) through this movie it is obvious that math is everywhere, and students should in fact care about them. It is accepted that, training one's brain in math allows them to see the connections between things otherwise missed, just as one of the three women, Johnson, does in *'Hidden Figures'*.

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Biographies

Picture 1: Katherine Johnson[1]

Picture 2: Mary Jackson[2]

Picture 3: Dorothy Johnson Vaughan [3]

Katherine Coleman Goble Johnson (born August 26, 1918 – February 24, 2020) was an African-American mathematician whose calculations of orbital mechanics as a NASA employee were critical to the success of the first and subsequent U.S. manned spaceflights. During her 35-year career at NASA and its predecessor, the National Advisory Committee for Aeronautics, she earned a reputation for mastering complex manual calculations and helped the space agency pioneer the use of computers to perform the tasks.

Johnson's work included calculating trajectories, launch windows and emergency return paths for Project Mercury spaceflights, including those of astronauts Alan Shepard, the first American in space, and John Glenn, the first American in orbit, and

[1] (Retrieved from: https://www.google.com/search?q=Katherine+Johnson&client=firefox-b-d&source=lnms&tbm=isch&sa=X&ved=0ahUKEwi_mL_0xOPiAhVQ_KQKHcSpB0EQ_AUIECgB&biw=1138&bih=527#imgrc=0jKkzCQCTbce$M:

[2] Retrieved from: https://www.google.com/search?q=Dorothy+Vaughan&client=firefox-b-d&source=lnms&tbm=isch&sa=X&ved=0ahUKEwjns5aZxuPiAhUoMewKHVNHCAYQ_AUIECgB&biw=1138&bih=527#imgrc=6SovmvarwMjx0M:)

[3] Retrieved from: https://www.google.com/search?q=Mary+Jackson&client=firefox-b-d&source=lnms&tbm=isch&sa=X&ved=0ahUKEwjX56KGx-PiAhWPbVAKHTL6Bf0Q_AUIECgB&biw=1138&bih=527#imgrc=pRLcd_QFW-XXhM:

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rendezvous paths for the Apollo lunar lander and command module on flights to the Moon. Her calculations were also essential to the beginning of the Space Shuttle program, and she worked on plans for a mission to Mars. In 2015, President Barack Obama awarded Johnson the Presidential Medal of Freedom.

Original source retrieved from Wikipedia: https://en.wikipedia.org/wiki/Katherine_Johnson

Dorothy Johnson Vaughan (September 20, 1910 – November 10, 2008) was an African American mathematician and human computer who worked for the National Advisory Committee for Aeronautics (NACA), and NASA, at Langley Research Center in Hampton, Virginia. In 1949, she became acting supervisor of the West Area Computers, the first African-American woman to supervise a group of staff at the center. She later was promoted officially to the position. During her 28-year career, Vaughan prepared for the introduction of machine computers in the early 1960s by teaching herself and her staff the programming language of FORTRAN; she later headed the programming section of the Analysis and Computation Division (ACD) at Langley.

Original source retrieved from Wikipedia: https://en.wikipedia.org/wiki/Dorothy_Vaughan

Mary Jackson (April 9, 1921 – February 11, 2005) was an African American mathematician and aerospace engineer at the National Advisory Committee for Aeronautics (NACA), which in 1958 was succeeded by the National Aeronautics and Space Administration (NASA). She worked at Langley Research Center in Hampton, Virginia, for most of her career. She started as a computer at the segregated West Area Computing division. She took advanced engineering classes and, in 1958, became NASA's first black female engineer. After 34 years at NASA, Jackson had earned the most senior engineering title available. She realized she could not earn further promotions without becoming a supervisor. She accepted a demotion to become a manager of both the Federal Women's Program, in the NASA Office of Equal Opportunity Programs, and of the Affirmative Action Program. In this role, she worked to influence both the hiring and promotion of women in NASA's science, engineering, and mathematics careers.

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Original source retrieved from Wikipedia: https://en.wikipedia.org/wiki/Mary_Jackson_(engineer)

Plot of the movie *'Hidden Figures'*

'*Hidden Figures*' the movie tells the incredible story of Katherine Jonson (actress Taraji P. Henson), Dorothy Vaughan (actress Octavia Spencer) and Mary Jackson (actress Janelle Monae) – three bright African-American women working at NASA, who served as the brains behind the launch into orbit of astronaut John Glenn, a spectacular achievement that turned around the Space Race. The visionary trio crossed all gender and racial lines and inspired generations.

The story unfolds at a time when the United States were competing Russia to put a man in space, NASA found idle talent in a group of African-American female mathematicians, the brains behind one of the greatest operations in U.S. history. Based on the incredibly true life stories of these three women, also known as 'human computers', we see how rapidly they raised the ranks of NASA alongside many of history's greatest minds purposely tasked with calculating the momentous launch of astronaut John Glenn into orbit, and guaranteeing his safe return. Dorothy Vaughan, Mary Jackson, and Katherine Gobels Johnson remained in history as true American Heroes with their brilliance and desire to dream big. They have accomplished something new for the human race.(Adjusted from -20th century Fox source: https://www.imdb.com/title/tt4846340/plotsummary)

The movie trailer:

You can view the trailer from: https://www.youtube.com/watch?v=5wfrDhgUMGI

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The Math behind the movie

 Use this link to get acquainted with the Mathematical part of the movie: https://www.youtube.com/watch?v=v-pbGAts_Fg

The initial idea was that to get America to the moon a bigger rocket was needed. Beyond that new kinds of trajectories were also needed because until then they were going up and hoping to succeed. Thus, new kinds of math were deemed important to model those trajectories. Of course many variables are involved in these kinds of equations, and tactually very difficult ones to estimate to the specifications both the military and astronauts were asking for—even the smallest error in the calculations as to when to slow the spacecraft down to start its descent into the atmosphere would have possibly result in burning it up, whilst the Navy demanded that NASA be able to bring the capsule down within a 20-mile square of ocean. Katherine Johnson was central to figuring out how to solve this problem, and in doing so exposes one of the reasons math is important and cool.

In the movie, she has an eureka moment while staring at a blackboard and realizes that "old math" might be the key. "She turns to Euler's method, which in layman's terms allows the mathematician to approximate a differential equation numerically without actually ever really solving it. It's a tedious process, but this process (and others that are philosophically similar to it) are what run our computers' math brains today—something Dorothy Vaughan helped to implement towards the end of the movie when she used the massive early IBM computer to start checking the numbers done by Johnson and other computers." [4]

[4] The Math Behind "Hidden Figures"- Why STEM is important and math is everywhere!, published 04/02/2017 Last accessed 25/07/2019. https://www.startalkradio.net/the-math-behind-hidden-figures-why-stem-is-important-and-math-is-everywhere/

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Johnson reveals the most fundamental reason why math is important to our lives, and that is because they are interconnected and intertwined in everything. Although Euler used that method in another context, here it was proven vital to be used to land men on the moon. Likewise, solving these differential equations isn't just important to rocket scientists—they have allowed for supreme advances in almost every field of study, from better patient care to self-driving cars.

Some formulas

The **Euler Method**, presented within the movie, is a numerical method to solve first order first degree differential equation with a given initial value. It is the most basic explicit method for numerical integration of ordinary differential equations. The deep understanding of the method is quite difficult and falls in the university math curricula. However, we can mention that is entirely based on the acquisition of special knowledge on 'derivatives'.

The **derivative of a function** of a real variable measures the sensitivity to change of the function value (output value) with respect to a change in its argument (input value). Derivatives are a fundamental tool of calculus. For example, the derivative of the position of a moving object with respect to time is the object's velocity: this measures how quickly the position of the object changes when time advances.

The derivative of a function of a single variable at a chosen input value, when it exists, is the **slope of the tangent line** to the graph of the function at that point (point P in the picture below). The tangent line is the best linear approximation of the function near that input value. For this reason, the derivative is often described as the "instantaneous rate of change", the ratio of the instantaneous change in the dependent variable to that of the independent variable.

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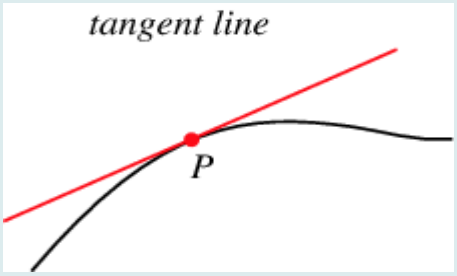
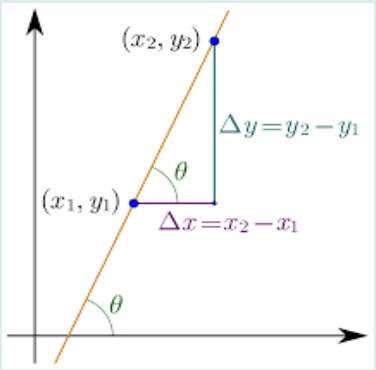


Picture 4: The derivative of the function at point P is the slope of the tangent line (in red) to the graph of the function (in black) at this specific point.

Accordingly, the derivative of the function is strongly interwoven with the formula that explains the slope.

$$Slope=\frac{change\ in\ Y}{change\ in\ X}=\frac{\Delta y}{\Delta x}=\frac{y2-y1}{x2-x1}$$



$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x)-f(x)}{\Delta x}$$

The formula occurs if we consider that there is a change in x-axis from point x to point (x+Δx), with a simultaneous change in y-axis, from f(x) to f(x+Δx).

The Greek letter Δ is being used when we want to show that this change is very small. The formula implies that we estimate this limit when Δx heads towards 0.

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Euler's method: Euler's method is a numerical method to solve first order first degree differential equation with a given initial value. It is the most basic explicit method for numerical integration of ordinary differential equations and is the simplest Runge–Kutta method. The Euler method is named after Leonhard Euler, who treated it in his book Institutionum calculi integralis (published 1768–1870).The Euler method is a first-order method, which means that the local error (error per step) is proportional to the square of the step size, and the global error (error at a given time) is proportional to the step size. The Euler method often serves as the basis to construct more complex methods, e.g., predictor–corrector method.

Picture 5: Katherine Johnson (played by Taraji P. Henson) calculates orbital insertion trajectories for the Mercury program using Euler's method in this scene from the movie Hidden Figures. [5]

[5] ***Credit: ™ and © 2017 Twentieth Century Fox Film Corporation. All rights reserved. Retrieved from:*** https://www.startalkradio.net/the-math-behind-hidden-figures-why-stem-is-important-and-math-is-everywhere/

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TASKS

TASK 1

As a lot of the math behind this movie is based on the work of one of math's most famous rock star, **Leonhard Euler**, make a small research and then a 5-7 slide presentation presenting this well-known math's personality.

TASK 2

Use the formula that estimates the derivative of a function in order to prove that the derivative of x^2 is 2x.

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LEARN MORE...

If you want to further investigate on the topics addressed in this tool, you may go through the following link:

20th Century Fox about the movie:

http://www.foxmovies.com/movies/hidden-figures

Trailer:

https://www.youtube.com/watch?v=5wfrDhgUMGI

Star Talk journal. Blog published on the 4th of February 2017 entitled 'The Math Behind "Hidden Figures" – Why STEM is important and math is everywhere:

https://www.startalkradio.net/the-math-behind-hidden-figures-why-stem-is-important-and-math-is-everywhere/

The Mathematical part of the movie:

https://www.youtube.com/watch?v=v-pbGAts_Fg

The biography of Mary Jackson:
https://en.wikipedia.org/wiki/Mary_Jackson_(engineer)

The biography of Dorothy Johnson Vaughan:

https://en.wikipedia.org/wiki/Dorothy_Vaughan

The biography of Katherine Johnson:

https://en.wikipedia.org/wiki/Katherine_Johnson

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