PART IV: Cinematography & Mathematics

AGE RANGE: 16-18

TOOL 35: BAYES THEOREM IN "BACK TO THE FUTURE" BY ROBERT ZEMECKIS

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Educator's Guide

Title: Bayes Theorem in the movie "Back to the Future" by Robert Zemeckis

Age Range: 16-18 years old

Duration: 2 hours

Mathematical Concepts: Probability, Bayes Theorem

Artistic Concepts: Fiction Genres, Science Fiction, Futurists, Theory of Relativity

General Objectives: To discover the mathematical concepts of Bayes Theorem by applying them to the movie and to acquire a more practical view of mathematics.

Instructions and Methodologies: Give the students the possibility to explore math by playing a scenario and watching the suggested videos. Your class will discover the different math concepts needed to learn probability.

Resources: This tool provides videos and online resources. The topics addressed in these resources will be an inspiration for you to find other materials to personalize and give nuance to your lesson.

Tips for the educator: Learning by doing is very efficient, especially for young learners with learning difficulties. Always explain the practical use of each math concept.

Desirable Outcomes and Competences: At the end of this tool, the student will be able to:

- o   Understand and use probabilities;
- o   Understand what a conditional probability is;
- o   Use Bayes' Theorem.

Debriefing and Evaluation:

Write 3 aspects you liked about this activity:	1. 2. 3.
Write 2 aspects that you have learned	1. 2.
Write 1 aspect for improvement	1.

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Introduction

Watching a movie can either be an active or a passive leisure activity. Movies can be valuable resources for learners to explore the different topics addressed. Some of them use mathematics in their plots, which students often don't really focus on though they will be more likely to understand a topic they heard about on TV.

Seeing the characters reflect on mathematical problems and concepts makes the viewer want to understand those concepts and solve those problems with them in the same way as they often try to guess the end of a movie, here they will learn new things just by following characters throughout the story.

Therefore, teaching students the mathematics that hide behind some movies can be a great added value to a math course, often considered too abstract, by giving learners a more practical and real-life sense of the possible uses of mathematics.

"Back to the Future" by Robert Zemeckis

Synopsis

Figure 1: Title of the movie

Back to the future is a trilogy that begins with the story of a teenager, Marty, who accidentally travels in time, ending up in 1955. He meets his parents when they were young and becomes his mother's crush. He will then need to make sure his parents fall in love with each other and eventually give birth to him. He is helped by a scientist called Dr. Emmett Lathrope.

🎥 Movie Trailer: https://www.youtube.com/watch?v=qvsgGtivCgs

Did you know that some of the technological innovations and features of our current society pictured in the second movie were quite close to our current reality?

🎥 Watch the following video by China Uncensored to see what they got right! https://www.youtube.com/watch?v=mV_Z3Zx0xls

🎥 You can also see what they got wrong in the following video: https://www.youtube.com/watch?v=xvWEIxdTB6Y

Back to the Future is a science fiction trilogy in which we can see how the authors pictured our modern world back in 1985. Science fiction is a genre that has had a great impact on society since it fosters scientific and technological innovation and thinking out of the box. It is connected to other genres such as horror or fantasy. Back to the Future was an inspiration for an animated science fiction by Justin Roiland and Dan Harmon for Cartoon Network that you might have heard of: Rick and Morty. If you look at the characters, you can easily see the similarities:

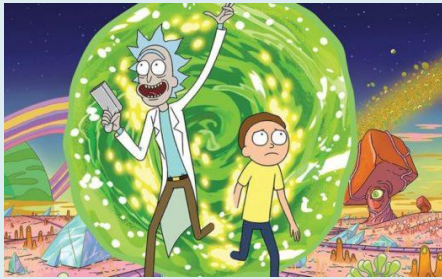**Figure 2: Marty and Dr. Emmett Lathrope in "Back to the future"[1]**

Figure 3: Rick and Morty in the cartoon called "Rick and Morty"[2]

🎥 You can watch this video to learn more about how science fiction helps us predict the future: https://www.youtube.com/watch?v=paXKoZ1pr5w

We know that time travel is often used in science fiction but how about reality? Would it be possible to travel in time?

Some scientists have elaborated on some theories about this topic. Do you want to learn about them?

🎥 You can watch this TED-Ed video:

https://www.youtube.com/watch?v=7H3ksmxwpWc

Glossary

Science Fiction: an artistic, literary, cinematographic, etc. genre that fosters scientific and technological innovation.

Futurist: person who tries to forecast the future.

Think tank: an organization that carries out research on some issues regarding politics, society, economy, etc.

Theory of Relativity: Albert Einstein's theory was a breakthrough in the fields of physics and astronomy in the 20th century and, amongst other things, presented time and space as an entity called spacetime.

Quantum mechanics: describes the world at the smallest possible scales of energy levels of atoms and subatomic particles.

Time dilation: comes from the theory of relativity and is a difference in the time measured by two observers, either because of a speed difference between them, or because they are located differently with respect to a gravitational field.

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[1] https://www.therakyatpost.com/2015/10/21/its-october-21-2015-the-day-marty-mcfly-came-back-to-the-future/
[2] http://popvinylworld.com/new-gamestop-exclusive-rick-and-morty-blips-chips-mystery-box-now-available-for-pre-order/

The math behind back to the future

Probability

As you might remember, probability can be calculated with different formulas. Let's have a quick reminder!

These are the definitions and formulas to remember:

Probability is used to get a more precise idea of the chances that something random has to happen.

It is written P(A) and will always be between 0 and 1.

Remember: **$0 \leq P(A) \leq 1$**

As two complementary events make up all the possibilities: **$P(A_C) + P(A) = 1$**

To calculate the probability of two events occurring at the same time, multiply their probabilities.

$P(A \cap B) = P(A) \times P(B)$

To calculate the probability of either one of two events happening, add their probabilities then take out the probability of both happening at the same time.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Conditional Probability

There are two types of events: independent events and dependent events.

Independent events are those that do not get affected by any other event, such as rolling a dice.

Dependent events are those that get affected by previous events, such as when you have a bag of candy and pick one red candy at the beginning. The probability of picking a red candy again will be different the second time.

This is called **conditional probability** and can be represented with a tree diagram.

Example: if we have ten candy among which you have **four blue** and **six red**:
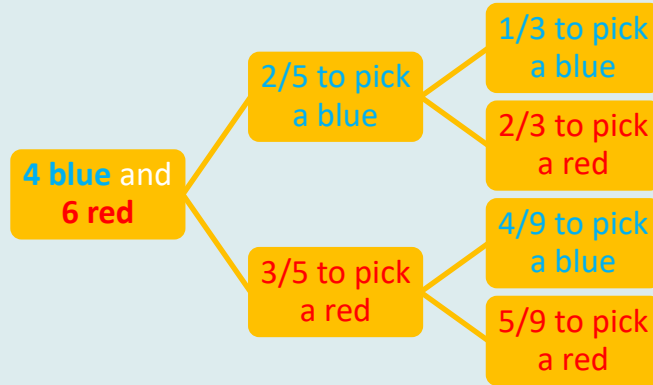
Let's see how to write this in mathematical notation:

- The event A is that we pick the **blue** candy
- The event B is that we pick the **red** one

P(A|B) means the probability of A happening given that B has happened.

We picked a **red** candy the first time. What is the probability to get a **blue** candy the second time?

$$P(B|A) = \frac{4}{9}$$

But how do we calculate that without using the tree diagram?

With Bayes' Theorem

Bayes' Theorem is used in order to find a conditional probability when you already know other probabilities.

The formula is as follows:

$$P(A|B) = \frac{P(A)\ P(B|A)}{P(B)}$$

In which:

P(A|B) is the probability of A happening given that B has happened

Let's use the movie for another example.

Imagine you have travelled in the DeLorean time machine.

It could have sent you either to 1861, 1955, 1985 or 2019, with equal chances.

$$P(1871) = \frac{1}{4} = \mathbf{0,25}$$

$$P(1955) = \frac{1}{4} = \mathbf{0,25}$$

$$P(1985) = \frac{1}{4} = \mathbf{0,25}$$

$$P(2019) = \frac{1}{4} = \mathbf{0,25}$$

You look around you and see a steam train!

This can only mean one thing: you are probably not in 2019!

Let's say there are only two types of trains: **steam** and **electric**.

We know that in 1871, **all** trains were steam, which means that:

P(Steam | 1871) = 1

In 1955, there were yet **less** steam trains.

Over time, this phenomenon went on until most trains became electric:

P(Steam | 1955) = 0,5

P(Steam | 1985) = 0,05

P(Steam | 2019) = 0,01

The probabilities have changed now that we have these figures!

Let's calculate the probability of being in year 2019 if we see a steam train:

$$P(2019 \mid Steam) = \frac{\mathbf{P(2019)\ P(Steam|2019)}}{\mathbf{P(Steam)}}$$

We still have to find out the probability of Steam in any year!

We know:

- P(2019) = 25% (0.25)
- P(Steam | 2019) = 1% (0.01)

Which means that:

- there is 25% of chances to be in 2019, in which case there is 1% of seeing a steam train.
- We can also deduce that there is 75% of chances to be in any other year.
- The probability of seeing a steam train in any other year is:

$$\frac{0,5+0,05+1}{3} = \mathbf{0,516}, \text{ or } 51,6\%.$$

P(Steam) = (0,25 * 0,01) + (0,75 * 0,516) = 0,39

Thus,

$$P(2019\,|\,Steam) = \frac{\mathbf{P(2019)\,P(Steam|2019)}}{\mathbf{P(Steam)}} = \frac{\mathbf{0,25 * 0,01}}{\mathbf{0,39}} = \frac{\mathbf{0,0025}}{\mathbf{0,39}} = \mathbf{0,0064}$$

$$P(1985\,|\,Steam) = \frac{\mathbf{P(1985)\,P(Steam|1985)}}{\mathbf{P(Steam)}} = \frac{\mathbf{0,25 * 0,05}}{\mathbf{0,39}} = \frac{\mathbf{0,0125}}{\mathbf{0,39}} = \mathbf{0,032}$$

$$P(1955\,|\,Steam) = \frac{\mathbf{P(1955)\,P(Steam|1955)}}{\mathbf{P(Steam)}} = \frac{\mathbf{0,25 * 0,5}}{\mathbf{0,39}} = \frac{\mathbf{0,125}}{\mathbf{0,39}} = \mathbf{0,32}$$

$$P(1871\,|\,Steam) = \frac{\mathbf{P(1871)\,P(Steam|1871)}}{\mathbf{P(Steam)}} = \frac{\mathbf{0,25 * 1}}{\mathbf{0,39}} = \frac{\mathbf{0,25}}{\mathbf{0,39}} = \mathbf{0,64}$$

You would think you have arrived to 1871, right?

However, you see a passer-by drinking from a bottle of Coca-Cola, which only exists since 1886.

You now know you're not in 1871 but you need to gather more information!

TASK

This task will help you use the movie's context to build your math skills by playing with probabilities.

Let's continue the previous exercise to find out in which year we are more precisely!

As you get out of the DeLorean, a car almost runs you over.

Here's what it looks like and what you know:

- It's a Ferrari 328

- Its production started **in 1985** and **12%** of the population owned one.

- **3%** of car collectors still drive them **in 2019**

Figure 4: Picture of a Ferrari 328

Calculate the P(1985|Ferrari328) and P(2019|Ferrari328) based on the new calculations.

Remember: P(1985) is no longer 0,25 but 0,032 and P(2019) has become 0,0064!

Use the answers to the previous part of the exercise in your calculations.

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TED-Ed video about how science-fiction can help predict the future:

https://www.youtube.com/watch?v=paXKoZ1pr5w

TED-Ed video on the possibility of time-travel:

https://www.youtube.com/watch?v=7H3ksmxwpWc

What the movie got right:

https://www.youtube.com/watch?v=mV_Z3Zx0xls

What they got wrong:

https://www.youtube.com/watch?v=xvWElxdTB6Y

TED-Ed video on Probability:

https://www.youtube.com/watch?v=IAiNqQi30-Y&t=159s

Other examples of Bayesian reasoning:

https://fs.blog/2018/09/bayes-theorem/