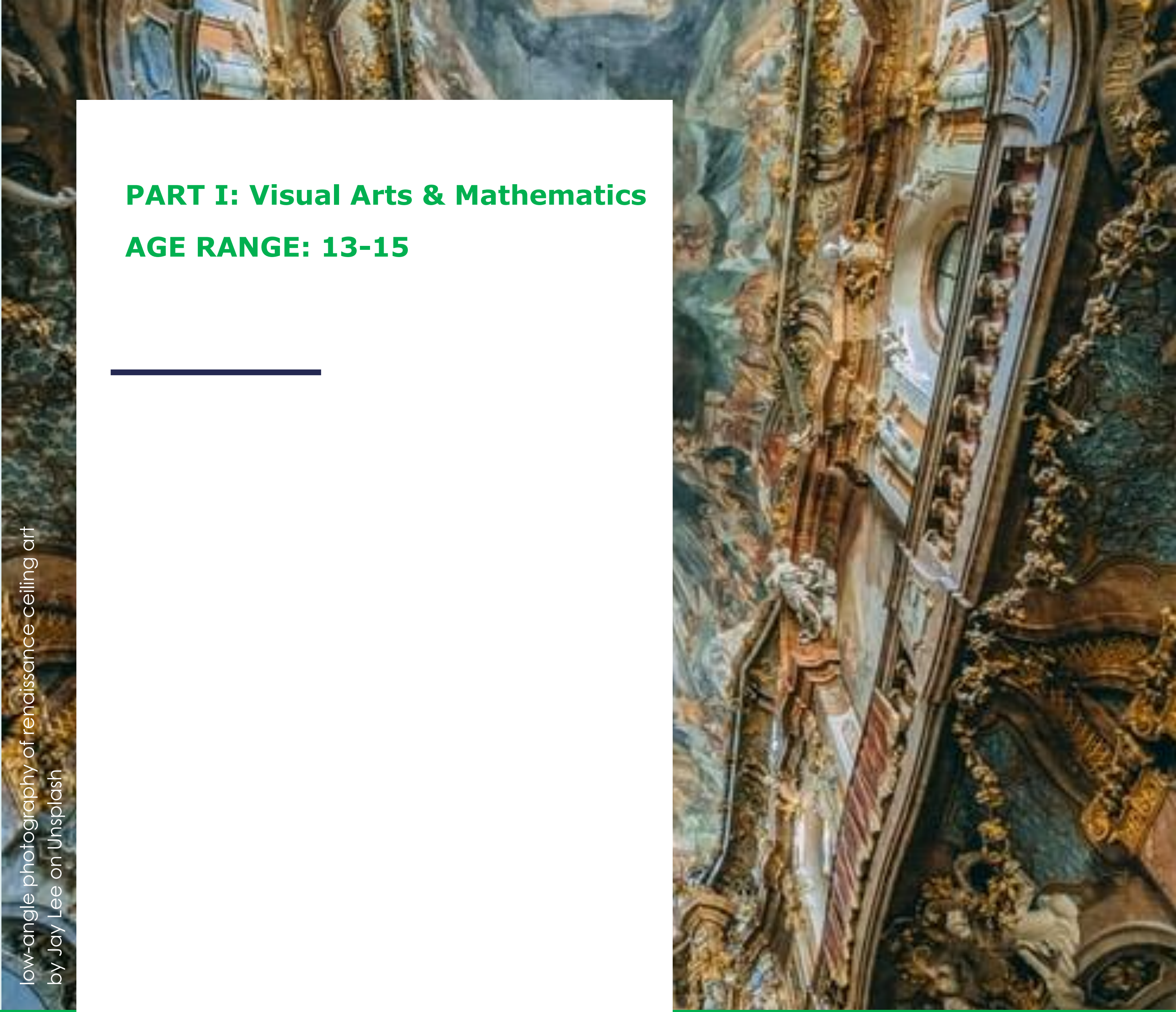

low-angle photography of renaissance ceiling art
by Jay Lee on Unsplash

PART I: Visual Arts & Mathematics

AGE RANGE: 13-15

TOOL 3: RENAISSANCE ART AND GEOMETRY

LogoPsyCom

TheArtOf Maths

Co-funded by the
Erasmus+ Programme
of the European Union

Educator's Guide

Title: Geometry in Renaissance Paintings and Architecture

Age Range: 13-15 years old

Duration: 2 hours

Mathematical Concepts: Perspective, Golden Ratio, Golden Rectangle, Polyhedra

Artistic Concepts: Renaissance Art, Linear Perspective, Vanishing Point, Foreshortening, Aerial Perspective, Focal Point, Classic and Ancient influence, Pediment

General Objectives: To discover the mathematical concepts hidden in Renaissance art, using perspective techniques and the Golden Ratio.

Instructions and Methodologies: The students will explore both fields as a whole, by drawing the art or watching the suggested videos that analyze famous Renaissance paintings. They will learn the basis of the mentioned math concepts.

Resources: This tool provides pictures and videos. The topics of the tool will be an inspiration for you to find other materials to personalize and nuance to your lesson.

Tips for the educator: Learning by doing is very efficient, especially for pupils with learning disorders. Encourage them to draw/paint using the learnt techniques.

Desirable Outcomes and Competences: At the end of this tool, the student will be able to:

- Understand the logical process behind the artists' use of linear and aerial perspective;
- Understand how the Golden Ratio is used in Renaissance Art;
- To recognize a platonic solid and know what constitutes a polyhedron.

Debriefing and Evaluation:

Write 3 aspects you liked about this activity:	1. 2. 3.
Write 2 aspects that you have learned	1. 2.
Write 1 aspect for improvement	1.

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Introduction

Mathematics and art seem to belong to two very different ways of thinking, respectively logic and creativity. As art is meant to express emotions and mathematics are used to express facts and reflection, one would think they have absolutely no connection.

However, many artists decided to study mathematics in their work. The improvement that geometric knowledge could bring to artistic creations made it an invaluable theoretical instrument in visual arts. Many artists from the Renaissance studied perspective, polyhedra and other mathematical concepts to reach a more life-like representation of the world.

Many scholars dealing with the history of art, have occasionally noted that the two greatest revolutions in the history of art, namely Renaissance and Modern Art, were made by artists who conceptualized new geometries; the perspective geometry for Renaissance and the multidimensional geometry for Modern Art.

In this tool, we will focus on Renaissance Art, its influences and the applicability of mathematics in artistic and architectural works. We will discuss the works of some renowned artists and architects who changed perspective and dimensions during the Renaissance.

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Renaissance Art

The Renaissance is the period between the Middle Ages and Modernity. It goes from the 14th to the 17th century, a time when artists were going back to the classical works of the Antiquity to find inspiration. The Renaissance gave birth to many renowned artists and architects. As they were inspired by the Antiquity, the historical period from the 8th century BC to the 6th century AD, some of the Renaissance works tend to be mistaken for Ancient art from Greece and the Roman Empire.

The emergence of Perspective

Before the 14th century, artists would represent important people as the tallest and most emphasized. However, this method did not illustrate the way the human eye sees things . This is why artists started to experiment with perspective, which consists of different lines and points that can represent what we see in 3D on a two-dimensional support.

Here is how the human eye sees things with perspective:

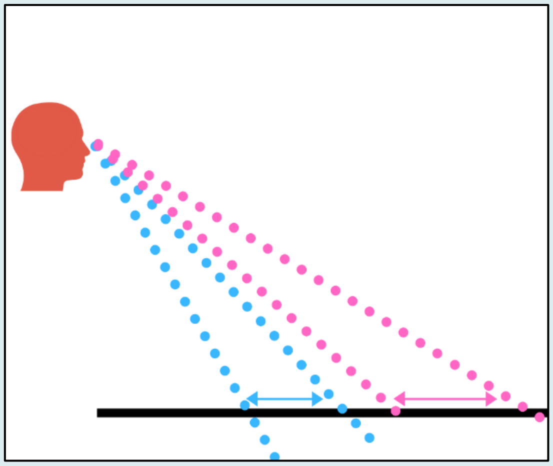

Figure 1: Lateral view of perspective

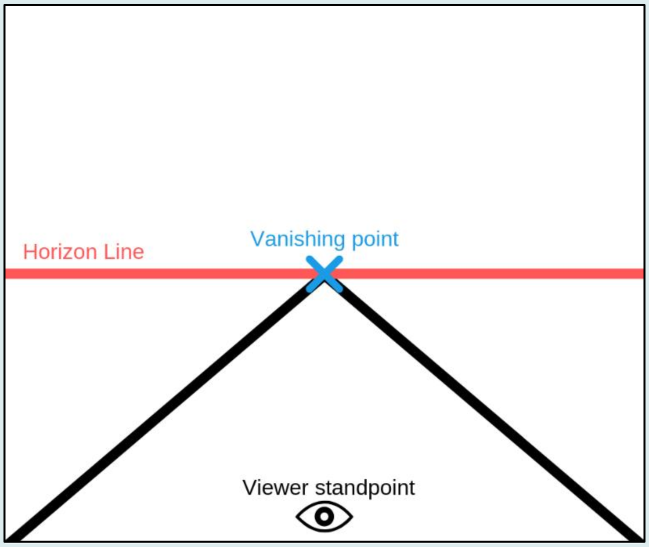

Figure 2: Front view of perspective

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With the studies on perspective, artists began to use different methods. Three main types of perspective were used widely in art works: linear perspective, foreshortening, and aerial perspective.

The **linear perspective** is built by the convergence of several lines on the picture. The point where they meet is called the **vanishing point**, which seems to be the furthest item on the image. Look at the lines on the second picture, they are a good imitation of the perspective lines created by the human eye.

Figure 1: Carlo Crivelli, The Annunciation with Saint Emidius,1486

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Foreshortening is the shortening of a first-plan item to give an impression of linear perspective. Here, you can see that the legs are very short and the torso very large. This is to give the viewer the impression of perspective, as if we were standing at the man's feet.

Figure 2: Mantegna Andrea, Dead Christ, 1470-71

Aerial or atmospheric perspective focuses on the painting's colors. When we see something from afar, it often seems very blurry, the colors are not very clear. Some painters created perspective by making the background of their painting blurrier and less contrasted than the foreground. This gives us the impression that we see the first-plan items very clearly and that the rest is too far away to see.

Figure 3: Leonardo da Vinci, Virgin of the rocks, 1503 – 1506

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Some examples of perspective

The Mona Lisa, one of Leonardo da Vinci's most famous paintings, also shows us his work on perspective. You can see that the colors and shapes are blurrier and more blended in the background, while the foreground is filled with details. This makes the Mona Lisa stand out and look even more intriguing for the viewer. As you will see later, this painting also contains the important math figure of the **Golden Rectangle**.

Figure 4: Leonardo da Vinci, Mona Lisa, 1503-1506

You probably recognize the name of **Leonardo da Vinci**, which has already been cited twice as an example. He was one of the most renowned paintors of the Renaissance.

The next painting is also his, but shows how he studied perpective very thoroughly before he started painting.

In the "Perspectival study of The Adoration of the magi", we can see the work he did on perspective for the painting's background. In this case, he drew all the lines that lead to the vanishing point in the background.

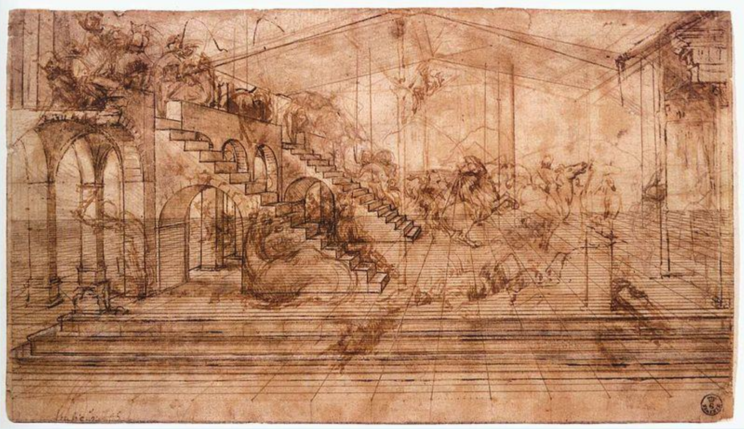

Figure 5: Perspectival study of The Adoration of the Magi, Leonardo da Vinci, ca.1481

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As you can see, the study of perspective focused on the background of this painting, which contains many details and thus uses linear perspective to show the background landscape, which seems to be much further away on the finished painting.

Figure 6: The Adoration of the Magi, Leonardo da Vinci, ca.1480

Dimensions and Proportions in Renaissance Art

In the Renaissance, the study of some ancient texts helped artists and architects develop new techniques to build their works in terms of proportion and dimension. Perspective was one of those techniques but there were many more.

Michel Angelo's David statue was initially meant to be on the roof of the Florence cathedral and thus seen from below, which is why its head and hands are so large. This is also an example of how Renaissance artists played with dimentions and proportions to create perspective and adapt to the viewer's standpoint.

Figure 7: Michel Angelo, David statue, 1501

You can learn more in the following video: https://www.youtube.com/watch?v=o9Kum_Jijdk ,

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Leonardo da Vinci was constantly studying science and mathematics in his works. The proportions of the two men in da Vinci's Vitruvius are based on the **Golden Ratio**. The Vitruvius man has been said to "combine the mathematics, religion, philosophy, architecture, and artistic skills of his age". Its proportions were believed to be the perfect proportions for the human body. The Mona Lisa (see picture above) also contains some golden rectangles.

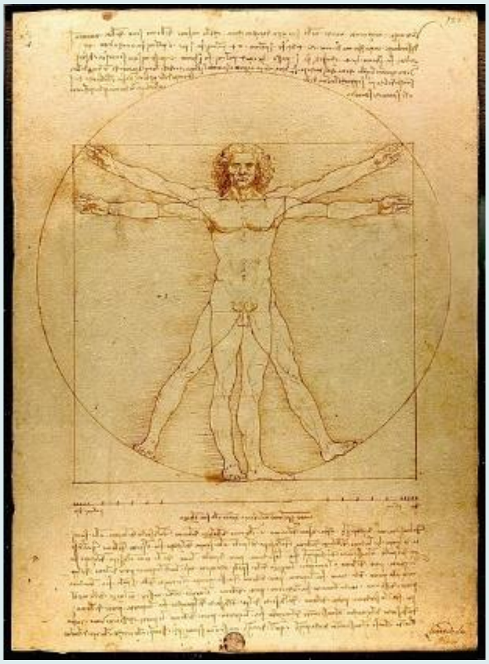

Figure 8: Leonardo da Vinci, The Vitruvian man, 1490

 Watch the following video to see how the golden ratio is used in this drawing: https://www.youtube.com/watch?v=gQ5a-RcLBuk.

Leon Battista Alberti designed part of the façade of Santa Maria Novella, in Florence. He was influenced by classicism and wanted to follow the patterns and perfect dimensions of the Antiquity. The church he had to work on was gothic, which made his work even more difficult. The result was a blended façade which reflected both gothic and classic influences. It shows the same geometric patterns and strategies described above to make its structure more orderly, using columns to define the edges of the building and finds inspiration in Greek temples with the **pediment** at the top of the façade.

Figure 9: Leon Battista Alberti, Santa Maria Novella, 1470

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Polyhedra in Renaissance Art

Polyhedra are geometric shapes that we can find in Renaissance art. Some of them are well-known and are called "Platonic solids" because artists from the Renaissance found inspiration in Plato's texts. Plato assigned a polyhedron to each of the elements. The Tetrahedron to Fire, the Icosahedron to Water, the Octahedron to Air, the Cube to Earth, and the Dodecahedron the "The Quinta Essentia" (the Universe). You will learn more in the mathematical part of the tool, but you can already notice that these solids are made of a combination of different shapes such as triangles and squares.

Figure 10: Plato's Association of the Platonic Solids to the elements, by Johannes Kepler

Figure 11: Uccello's Untitled mosaic,1425

Paolo Uccello's Untitled mosaic in the Basilica of San Marco, in Venice (1425 – 1430), is one of the representations of the Renaissance use of polyhedra in artistic compositions. You can see that he drew different solids on this tile, one of them showing a very complex polyhedron, which could be decomposed in several pieces. The use of polyhedra in this case helps the artist give us an illusion of depth and third dimension.

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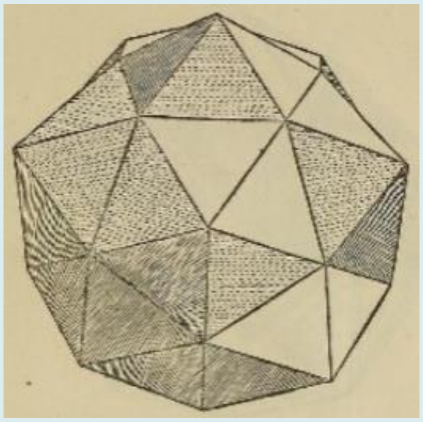

Figure 12: Leonardo da Vinci's illustration in "De Divina Proportione", 1509

Leonardo da Vinci also studied polyhedra and drew the illustration of a book called "De Divina Proportione" (On Divine Proportion). You can see the example on the left as one of the cases he studied. The artist used perspective to draw the different shapes and combined them to make a solid. Again, this technique creates an illusion of depth. We get the impression that if we could touch the shape, we could turn it around and see all its sides in three dimensions.

In ancient architecture, the most representative examples of polyhedra are the Egyptian pyramids.

Figure 13:Pyramids of Giza, Egypt

However, during the Renaissance, polyhedra were mostly used in art and it is not until the modern times that architects started to really explore the possibilities they offer in their buildings.

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Glossary

Linear Perspective: the painting technique that uses lines and a vanishing point to create perspective.

Foreshortening: the painting technique that shortens a foreground item to create an illusion of perspective.

Aerial Perspective: the painting technique that blurs the background items to create an illusion of distance and emphasize the foreground items.

Vanishing Point: in a painting, the vanishing point is the convergence of the perspective lines, which creates the illusion of a three-dimensional painting.

Focal Point: the painting's item that is emphasized to draw the attention of the viewers.

Pediment: is the triangular element often found on temples in Ancient architecture.

Gothic: a type of architecture that was very present in Europe from the 12th to the 16th century. A famous example is the Notre-Dame cathedral in Paris. You can learn about it in the tool about gothic art.

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The Golden number:

The Golden number is a rather unique number in mathematics. It is approximately 1,618 and is often used in art and architecture. We use the Greek letter φ (phi) to refer to it.

The golden ratio is the use we make of this number in different disciplines.
Imagine we cut a line in two different parts: **a** and **b**. When we use the golden ratio, the whole length divided by the long part is equal to the long part divided by the short one.

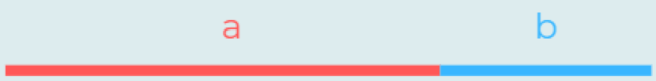

Figure 14: Line divided according to the Golden Ratio

To make it short, remember this formula:

$$\varphi = \frac{\mathbf{a+b}}{\mathbf{a}} = \frac{\mathbf{a}}{\mathbf{b}} = \mathbf{1,618}$$

The golden ratio can then be applied to a rectangle, called the **Golden rectangle**. As it was seen as the most perfect shape, many Renaissance artists and architects used it in their work.

As we have done it with the line ab, let's divide a **rectangle AB** into 2 different parts: a **square A** and a **rectangle B** in which:

- all sides of square A have a length of **a**;
- the long sides of rectangle B have a length of **a**;
- the short sides of rectangle B a length of **b**.

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Look at the following picture:

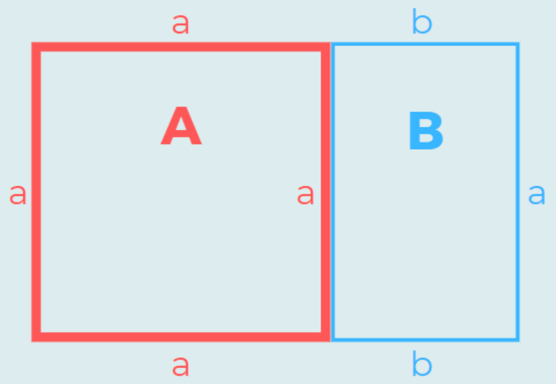

Figure 15: Rectangle divided according to the Golden Ratio

To have the perfect rectangle, we will use the same formula. Imagine for instance that the square **A** is 2cm x 2cm. If we want to find the side **b**:

- We know that $\frac{a}{b} = 1{,}618$
- We also know that a = 2
- We can say that $\frac{2}{b} = 1{,}618$
- And that 2 = b * 1,618
- If we isolate b, we have: b = $\frac{2}{1{,}618}$
- So, b = 1,236

Let's check the result using both formulas:

- $\frac{2 + 1{,}236}{2} = 1.618$
- $\frac{2}{1.236} = 1{,}618$

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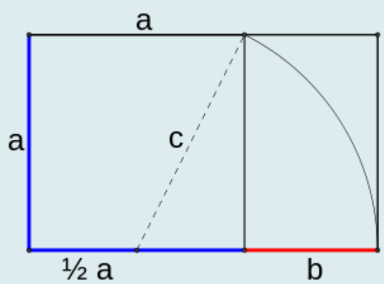
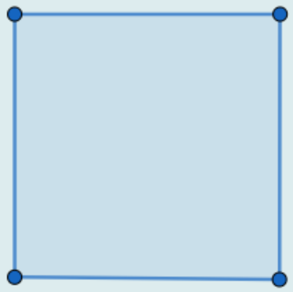
You can also use a compass and ruler to draw the golden rectangle:



1. Place your compass' needle point in the middle of the bottom side: **½ a**
2. Open your compass to touch the opposite angle
3. Draw a curve from the prolongation of the bottom side to its opposite angle
4. Draw the rectangle B from the start of the curve to the prolongation of the top and bottom sides of square A



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Polyhedra

A Polyhedron is a solid figure made of flat surfaces called polygons. These surfaces cannot be rounded nor curved.

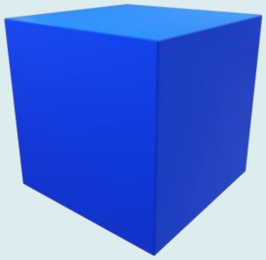
To check if a solid is a convex polyhedron, you can use **Euler's polyhedral formula**:

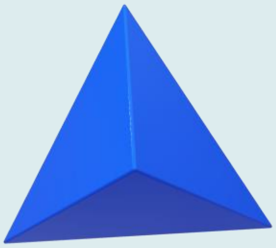
$$F + V - E = 2$$

in which **F** is the number of **Faces**, **V** the number of **Vertices**, and **E** is the number of **Edges**.

Let's use it in some examples:

	The cube	**The Tetrahedron**
F	6	4
V	8	4
E	12	6
F + V – E = 2	6 + 8 – 12 = 2	4 + 4 – 6 = 2





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TASK

These tasks will enable you to comprehend the ways in which perspective, the golden ratio and polyhedra were used in Renaissance Art.

A) Perspective Lines

- Draw the perspective lines on the second picture below
- Draw the vanishing point at the convergence of the perspective lines.

Look at how we have done it in Piero della Francesca's "The ideal city":

Figure 14: Piero della Francesca, The ideal city, 1470

Figure 15: Leonardo da Vinci, The last supper, 1495

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B) Solve a math problem about architecture using the golden ratio

Figure 16: The Parthenon, Greece, 5th century BC

See this temple? It's called the Parthenon, an ancient Greek temple with the perfect dimensions of the Golden ratio. It has been an inspiration to many architects of the Renaissance.

Let's draw a Temple using the formula.

- On scale, you know that the **square A has sides of 2,7 cm**.
- Calculate the **side b** of the rectangle.
- Draw the rectangle with the **compass method** you just learnt.
- Decorate your Golden rectangle with renaissance features (columns, a pediment, etc.).

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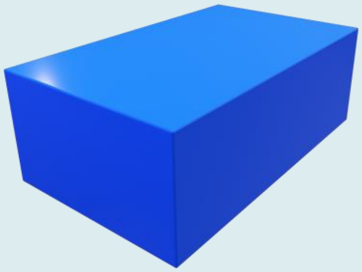
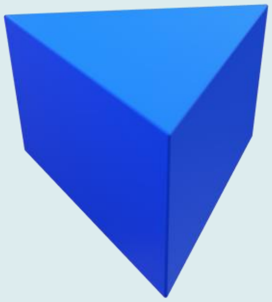
You can also use the online graphing tool **GeoGebra** to draw the shapes more precisely if you want!

C) Find out if the following shapes are convex polyhedra:





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LEARN MORE…

History of the Renaissance:

https://www.youtube.com/watch?v=Vufba_ZcoR0

Analysis of Michelangelo's statue of David:

https://www.youtube.com/watch?v=o9Kum_Jijdk

Leonardo da Vinci's Vitruvius:

https://www.youtube.com/watch?v=gQ5a-RcLBuk (Leonardo da Vinci's Vitruvius)

Leonardo da Vinci's Vitruvius)

https://www.youtube.com/watch?v=aMsaFP3kgqQ

Animated Polyhedra examples:

https://www.mathsisfun.com/geometry/polyhedron-models.html?m=Tetrahedron

A description of Alberti's Santa Maria Novella:

https://www.khanacademy.org/humanities/renaissance-reformation/early-renaissance1/sculpture-architecture-florence/v/alberti-santa-maria-novella

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