PART II: Music & Mathematics

AGE RANGE: 13-15

TOOL 17: MUSIC AND THE GOLDEN RATIO

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TheArtOf Maths

Educator's Guide

Title: Music and the Golden Ratio

Age Range: 13-15 years old

Duration: 2 hours

Mathematical Concepts: Golden Number, Golden Ratio, Fibonacci Sequence

Artistic Concepts: Ancient Greek Music, Muses, Harmony

General Objectives: To discover the mathematical concepts hidden in musical compositions and acquire a more practical view of the use of math.

Instructions and Methodologies: The students will explore both fields as a whole, by listening to the music or playing it by watching the suggested videos that analyze musical compositions. They will discover the basis of the mentioned math concepts.

Resources: This tool provides videos for you to use in your classroom. The topics addressed in the tool will help you find other materials to personalize and give nuance to your lesson.

Tips for the educator: Learning by doing is very efficient, especially with young pupils with learning difficulties. Always explain the practical use of each math concept.

Desirable Outcomes and Competences: At the end of this tool, the student will be able to:

- o Understand the logical process behind music composition;
- o Understand the use of the Golden Ratio in music;
- o To use the Golden ratio to find the climax of a musical composition.

Debriefing and Evaluation:

Write 3 aspects you liked about this activity:	1. 2. 3.
Write 2 aspects that you have learned	1. 2.
Write 1 aspect for improvement	1.

Introduction

Music and mathematics don't show an obvious connection for those who have never composed or read a music sheet. However, it appears clearly that the timing of musical compositions and the structuring of the sheet by measures evokes a mathematical way of thinking.

Many scholars have studied the implication of mathematics in the arts. Music was one of the focus points of their studies and it was found that, throughout history, many mathematicians had explored that question. Pythagoras, Leonardo Bonacci (also know as Fibonacci), and many others have contributed to the research. Different aspects of mathematics, ranging from basic geometry and number sequences to trigonometry, have shown to be used in musical compositions.

Within this tool, we will focus on the applicability of mathematics in musical compositions by exploring the options the Golden ratio offers for music composition.

The divine proportion in music

The divine proportion, or Golden Ratio (written φ), is considered as the most perfect proportion and can be observed in many different fields such as art, nature, and even music. Some would argue that it is rather unlikely that all artists and musicians built their work around this particular number. However, intentionally or not, this number has been included in many artistic and musical compositions.

The Fibonacci Sequence, which is closely linked to the Golden Ratio, can also be observed in many works of art and in nature.

Here are some visual examples:

- The sunflower contains Fibonacci's Sequence;
- The Greek temple has the divine proportions of the golden rectangle.
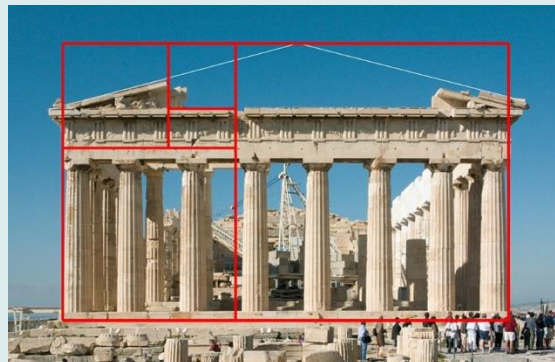

Figure 1: Sunflower with Fibonacci Sequence

Figure 2: Greek Temple with divine proportions [1]

You can check this video to learn more about how Fibonacci's Sequence is represented in nature: https://www.youtube.com/watch?v=_GkxCIW46to.

The Fibonacci Sequence is based on the perfect proportions of the Golden Ratio, which makes it a great inspiration for musicians to experiment in their compositions. As it is used in art and architecture to define the perfect dimensions, the Golden Ratio can also be used in musical composition to determine the climax of the melody. The

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[1] Image retrieved from : https://www.telitec.com/2019/05/27/golden-ratio/

climax will then be placed where the ratio of the total amount of bars to the climax bar reaches φ.

🎥 The musicians in this video try to compose based on the Fibonacci Sequence and the Golden Ratio: https://www.youtube.com/watch?v=9mozmHgg9Sk

You will find many different interpretations of how the Golden Ratio and the Fibonacci Sequence can be used in musical compositions. Some decide to play a number of notes corresponding to the Fibonacci numbers, some will assign those numbers to their piano's tiles, etc.

Glossary

Divine Proportion: is the same as the Golden Ratio, and is called divine proportion because of its inexplicable presence around us.

Fibonacci (ca. 1170-1240): Leonardo Bonacci was an Italian mathematician who created a famous sequence linked to the Divine Proportion.

Climax: the culmination of something, the most exciting point of something.

Bar: or measure, which is the division of the music sheet

The math behind music composition

The Golden Ratio:

The Golden number is a rather unique number in mathematics. It is approximately **1,618** and is often used in art, music, architecture, etc.

We use the Greek letter **φ (phi)** to refer to it.

The golden ratio is the use we make of this number in different disciplines.

Imagine we cut a line in **two** different parts a and b. When we use the golden ratio, the **whole length** divided by the **long side** is equal to the **long side** divided by the **short side**.

Figure 3: Line divided according to the Golden Ratio

To make it short, remember this formula:

$$\varphi = \frac{(a+b)}{a} = \frac{a}{b} = 1,618$$

You know that musical compositions are written on music sheets. These music sheets are divided in bars or measures. In those, you will find the notes to play. In order to notice the Golden Ratio in the musical compositions you hear, you will need to use this formula.

Here are some steps to follow:

Imagine the music sheet is this line and you want to apply that ratio to it.

- Count the number of bars
- Take the total number of bars = (a+b)
- Divide it by φ to find a
- Divide a by φ to find b

TASK

This task will enable you to use the math you have learnt about the Golden ratio in a musical composition.

Let's hear it!

Do you know who Claude Debussy is? He was a very well-known French composer who lived from 1862 to 1918. He was one of the first composers to use impressionism in his work. It seems he has nothing to do with mathematics. However, some of his compositions clearly show a focus on the use of the Golden Ratio.
" Dialogue du vent et de la mer" is one of them.

🎵   You can hear it here: https://www.youtube.com/watch?v=aIE3sOwEu3g

Here is what you know: The introduction is **55-bars long**

Complete the following information:

1. **Find a**

2. **Find b**

3. **Verify your results**

4. **Draw a line to represent this ratio by dividing the dimensions by 10**

LEARN MORE...

Video about the use of Fibonacci and the Golden ratio in music composition:

https://www.youtube.com/watch?v=9mozmHgg9Sk

Learn how Fibonacci and the Golden ratio are represented in nature:

https://www.youtube.com/watch?v=_GkxCIW46to

How to compose a song with Fibonacci's Sequence and the Golden Ratio:

http://www.faena.com/aleph/articles/how-to-compose-a-song-with-the-golden-ratio-and-the-fibonacci-sequence/

Paper on the mathematical study of music:

http://eprints.ma.man.ac.uk/1548/1/covered/MIMS_ep2010_103.pdf

More examples of the use of the Golden Ratio in music:

https://www.cmuse.org/classical-pieces-with-the-golden-ratio/

Video about how composers use the Golden Ratio and Fibonacci:

https://www.youtube.com/watch?v=yAyi8e5RDXw