

## PART I: Visual Arts \&

 Mathematics AGE RANGE: 16-18
## TOOL 12: FIBONACCI SPIRAL

 IN VISUAL ARTS
## (14) Thathot

$\stackrel{\star^{\star *}{ }_{\star}^{*}}{\stackrel{\star}{*}} \quad$| Co-funded by the |
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| Erasmus+ Programme |
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## Educator's Guide

Title: Fibonacci spiral in visual arts
Age Range: 16-18 years old
Duration: 2 hours
Mathematical Concepts: Fibonacci sequence
Artistic Concepts: Line of beauty
General Objectives: This task will make you learn more about Fibonacci sequences. It will show you how it can be seen in nature and thereby be replicated in arts Resources: This tool provides pictures for you to use in your classroom. The topics addressed in these resources will also be an inspiration for you to find other materials that you might find relevant in order to personalize and give nuance to your lesson. Tips for the educator: Learning by doing has proven to be very efficient, especially with young learners with lower attention span and learning difficulties. Don't forget to always explain what each math concept is useful for, practically.

Desirable Outcomes and Competences: The methods and activities proposed will help your students understand the idea of Fibonacci sequence It will introduce the topic more practically and will give them the opportunity to place the math concepts learnt in real-life applications as well as arts applications.

Debriefing and Evaluation:

| Write 3 aspects you liked about this | 1. |
| :--- | :--- |
| activity: | 2. |
|  | 3. |
| Write 2 aspects that you have learned | 1. |
|  | 2. |
| Write 1 aspect for improvement | 1. |

# Erasmus+ Introduction 

Watch these films:
https://www.youtube.com/watch?reload=9\&v=iEnR8zupK0A
https://www.youtube.com/watch? $\mathrm{v}=\mathrm{wTlw} 7 \mathrm{fN}$ cO-0
"The nine Indian numbers are: 987654321 and with these nine digits, and with the character 0 ... all numbers can be written."

With these words, Leonardo Fibonacci begins his book Liber Abaci, 1202. It is the first book that introduces the Arabic numerals in Europe! Fibonacci's father was a trader who for some time lived with his family in North Africa. It was there that Fibonacci began to take an interest in mathematics. He learned to use the Arabic numerals, which is the basis of the figures we use today by his Arabic teachers. They were much easier to count on than the Roman figures we still used in Europe. Fibonacci worked in Pisa in the 13th century and is regarded as one of the most important mathematicians.

## The Fibonacci sequence

The sequence named after Fibonacci originally comes from the following problem: At the beginning of a year, in a cage there is a newborn rabbit couple (a male and a female). A rabbit couple can get babies after two months and then give birth to a female baby rabbit and a male rabbit baby with a monthly interval, all of whom continue to multiply in the same way as above. How many rabbit pairs are there after a year just after the last rabbit pairs have been born?

In the beginning of the first month there is $M(1)=1$ rabbit pair. Since it took a month before they could reproduce, it is also $2 M(2)=1$ rabbit pair in the beginning of the month. Month three, the rabbit couple will have a new rabbit pair, $M(3)=2$ rabbit pairs. In the next month, the original rabbit couple will get a new baby couple, while the other rabbit couple will not get anything this month, $M(4)=3$ rabbit pairs. The five months the two first rabbit pairs give birth to new rabbit pairs, hence $M(5)=5$ rabbit pairs, $M(6)=8$ rabbit pairs, $M(7)=13$ rabbit pairs and so on.

In the sequence you get the next digit by adding the two previous digits, $3+5=8$, $5+8=13 \ldots$

In nature you can find the Fibonacci numbers in different contexts. Looking at a spruce or pine cone from the base (the attachment point), the cone's scales form spirals both clockwise and counterclockwise. If the cone is unbroken, the number of spirals in the cone is equal to the Fibonacci numbers 5,8 or 13.

The seeds in a sunflower form spirals clockwise and counterclockwise and the number of spirals can be equal to the Fibonacci numbers 34, 55, 89, 144 and even 233.


Picture 1: Sunflower https://www.needpix.com/photo/420789/sunflower-seeds-center-nature-sun-summer-grow-garden-yellow

If you draw the sequence as squares, you can draw the Fibonacci spiral, this spiral can also be found in nature!


Picture 2 Fibonacci blocks https://commons.wikimedia.org/wiki/File:FibonacciBlocks.svg Picture 3 Fibonacci spiral https://commons.wikimedia.org/wiki/File:Fibonacci spiral 13.svg

For example, in spiral shells and in galaxies!


Picture 4 https://en.wikipedia.org/wiki/File:NautilusCutawayLogarithmicSpiral.jpg
Picture 5 https://www.flickr.com/photos/gsfc/14172908657

## The Math behind the Fibonacci sequence

Fibonacci's sequence is an example of a recursive sequence. A recursive sequence is a sequence of numbers where each number can be calculated using one or more of the preceding numbers. If $F_{n}$ is the $n$ :th Fibonaccital we have:

$$
F_{n+2}=F_{n+1}+F_{n}
$$

We also have two starting values:

$$
F_{1}=1 \text { and } F_{2}=1
$$

There is another sequence called the Lucas sequence, which is defined by the same recursion formula such as the Fibonacci sequence,

$$
L_{n+2}=L_{n+1}+L_{n}
$$

But the Lucas sequence initial values differ from the Fibonacci sequence, they look like this:

$$
L_{1}=1 \text { and } L_{2}=3
$$

Thus, the first numbers in the Lukas sequence are: $1,3,4,7,11,18,29 \ldots$

## TASKS

TASK 1

## Read The Rabbit problem again

1. How many rabbit pairs are there at the beginning of the seventh month? ( $M=7$ )
2. How many rabbit pairs are there after a year just after the first rabbit pairs have been born? ( $\mathrm{M}=12$ )
3. What is $M(25)$, when $M(21)=10946$ and $M(23)=28657$ ?

## TASK 2

## The Staircase problem again

You can go up a flight of stairs in a way that you have to climb the first step (step 1), but from here you can choose to take one or two steps at the time. So the next step you take will take you to the second or the third step in the stair. How many different ways can you go up the flight of stairs, if the amount of steps are: a) 3 , b) 4, c) 10 , d) 20 ?

You can solve the Staircase problem digitally with, for example, Python or Java. For 20 steps it's almost necessary...

## LEARN MORE...

A further explanation about the Fibonacci sequence and the Golden Ratio. https://www.mathsisfun.com/numbers/fibonacci-sequence.html

Wikipedia on Fibonacci sequence.
https://en.wikipedia.org/wiki/Fibonacci number

Coding the staircase problem.
https://www.dailycodingproblem.com/blog/staircase-problem/

Coding the staircase problem.
https://www.geeksforgeeks.org/count-ways-reach-nth-stair/

