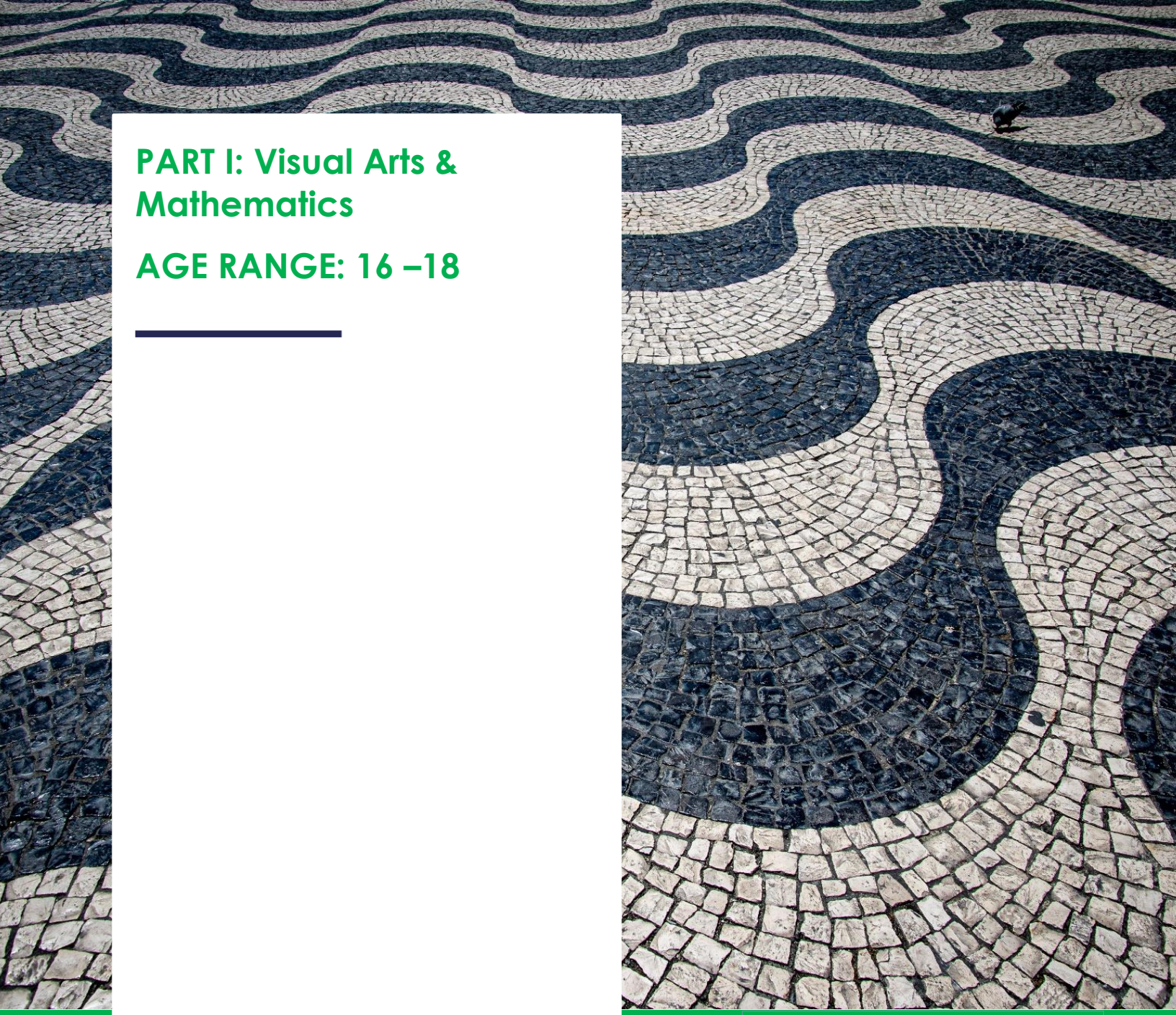

"Cobblestone Floor"
(Source: Photo by Magda Ehlers from Pexels)

PART I: Visual Arts & Mathematics

AGE RANGE: 16 –18

TOOL 10: PATTERNS IN THE PORTUGUESE PAVEMENT

SPEL – Sociedade Promotora de Estabelecimentos de Ensino

TheArtOf Maths

Co-funded by the Erasmus+ Programme of the European Union

Educator's Guide

Title: Patterns in the Portuguese Pavement

Age range: 16 –18 years old

Duration: 2 hours

Mathematical concepts: Isometries and Patterns

Artistic concepts: Portuguese Pavement and Patterns

General objective: Identify the different types of symmetry and patterns

Instructions and Methodologies: In addition to the explanation of the theoretical concepts, it is important to use videos and elaborate symmetries with Geogebra for a better understanding of the contents

Resources: Computer with an internet connection; Access to the website: https://www.geogebra.org/m/KGWhcAqc

Tips for the educator: Start by explaining the theoretical concepts and, if possible, use images and videos for an effective explanation. Demonstrate/elaborate a few symmetries and/or patterns with Geogebra for a better understanding of the contents and to allow that students solve the exercises by themselves

Learning Outcomes and Competencies:

At the end of this module, the student will be able to:

- Identify and apply several types of symmetries;
- Identify and elaborate several types of patterns

Debriefing and Evaluation:

Write 3 aspects you liked about this activity:	1. 2. 3.
Write 2 aspects that you have learned	1. 2.
Write 1 aspect for improvement	1.

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Introduction

Many public and private pavements of the Portuguese cities are covered by a layer of small stones, usually, black and white limestone of irregular shape, forming decorative patterns or mosaics through colour contrast. This type of coating is called Portuguese pavement.

Unintentionally, we are stepping on Mathematics, as in this type of pavement, we can easily find symmetries, reflexions, rotations, and translations. The mathematicians call to the repetition of drawings, in one or more directions, friezes or wallpaper patterns, respectively. In the Portuguese pavements, there are examples of rosettes and flat figures in which the number of symmetries is limited (rotations or reflexions). Additionally, there is a fixed point in the plane for all the symmetries in the figure.

There are only 7 distinct types of friezes and 17 different types of patterns. In Lisbon, possibly the city where the Portuguese pavement has emerged, it is possible to find all the 7 friezes and 12 of the 17 patterns. In the Azores, there are two cities in which the 7 types of friezes are identified, which are Ponta Delgada and Angra do Heroísmo, which achieved the designation of "City of the Seven Friezes."

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Patterns in the Portuguese Pavement

Fig. 1 – Portuguese pavement
(Source: Photo by Silas Camargo Silão from Pixabay. Retrieved from: Retirado de: https://pixabay.com/pt/photos/cal%C3%A7ada-piso-ch%C3%A3o-pedra-1842527/(11/07/2019))

In many Portuguese cities, the streets and pavements are still covered by a layer of small stones, usually, black and white limestone of irregular shape, forming decorative patterns or mosaics through colour contrast. This type of coating is called Portuguese pavement.

This type of pavements might have emerged in Portugal around 1500, but the Portuguese pavements, in other words, the decorative pavement as we know today, might only have appeared in the mid-19th century.

Back then, the Lieutenant-General and Engineer, Eusébio Furtado, guided the construction of the pavement of the military quarter from the Hunters Battalion No. 5, in the Castle of Saint Jorge, using the labour of the prisoners from the Prison of Limoeiro. Consequently, the first decorative floor emerged, which presented a zigzag of great visual impact.

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After the success of this pavement, all the area from the Praça do Rossio (nearly 9000 m^2) was transformed into Portuguese pavement, followed by many other pavements in Lisbon. After a few years, other Portuguese cities transformed their pavements into Portuguese pavements, followed by other Portuguese-speaking countries and, later on, other countries.

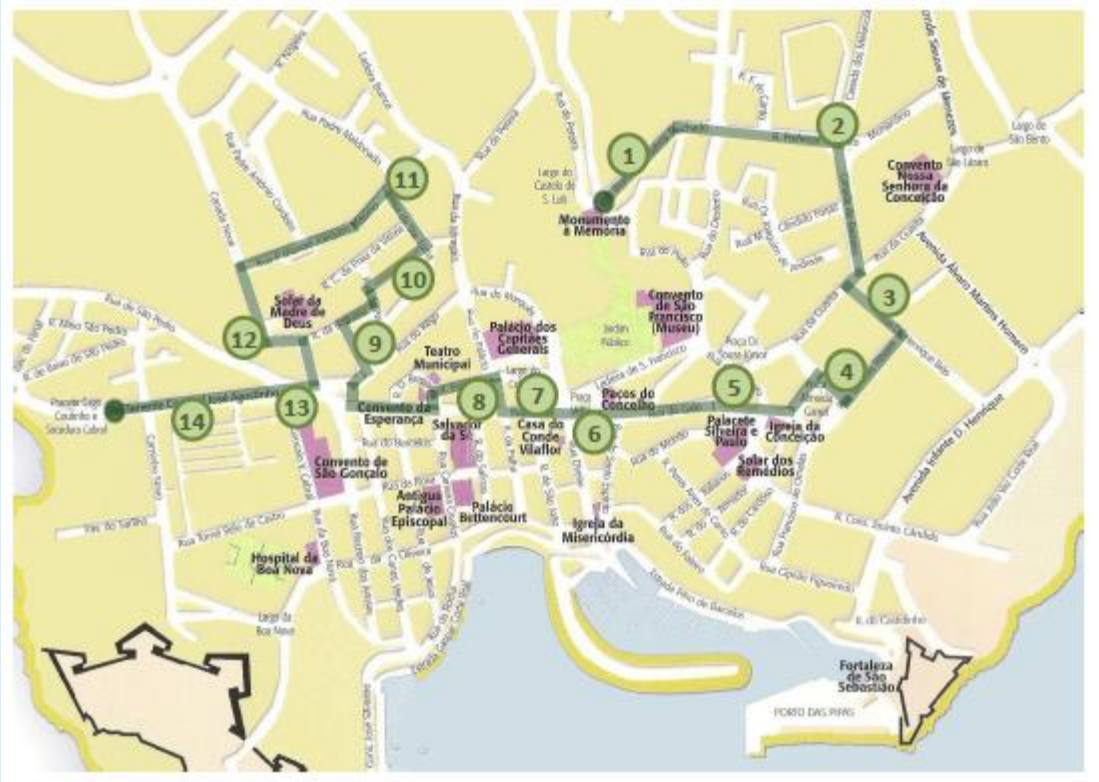

Fig. 2 – Friezes tour of the city of Angra do Heroísmo
(Source: Teixeira, R. C. (2013). *Roteiro de Frisos da Cidade de Angra do Heroísmo*. Retrieved from: https://www.mat.uc.pt/mpt2013/files/Roteiro-de-frisos-Angra.pdf (11.07.2019))

The workers specialised in the placement of this type of pavement are called master craftsmen and, in their work, is common to find themes as the "Mar Largo" (wide sea), used in the Praça do Rossio and in the Copacabana pavement, as well as dolphins, caravels, flowers, etc.

The Portuguese pavement is very rich in mathematical concepts as symmetries, reflexions, rotations, and translations. It is frequent to find rosettes, friezes, and patterns in these pavements.

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Glossary

Portuguese pavement: pavement that results from the coating with stones, usually, with black and white limestone of irregular shape, forming decorative patterns or mosaics through colour contrast.

Limestone: type of sedimentary rock.

"Mar largo" (Wide sea): pattern imitating the rhythm of the tides and waves.

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The Math behind the Patterns in the Portuguese Pavement

The Portuguese pavement is rich in rosettes, patterns, and friezes, thus, before we address these concepts, it is necessary to make a small review regarding isometries.

An **isometry** is a geometric transformation that preserves the distances between points, which means that the initial figure and its transformed are congruent.

The only isometries of the plane are **translations**, **rotations**, **reflexions**, and **glide reflexions**.

When, through an isometry different from the identity, the image of a figure matches with the original figure, then this figure has symmetry. There is a symmetry for each one of the four isometries.

Types of symmetry:
A flat figure can have:

- **Symmetry of reflexion**, if there is a reflexion that makes it invariant;
- **Symmetry of rotation**, if there is a rotation that makes it invariant;
- **Symmetry of translation**, if there is a translation that makes it invariant;
- **Symmetry of glide reflexion**, if there is a reflexion that makes it invariant.

Now, our studies will focus on **friezes**.

A **frieze** is a flat figure that has an infinity of translation symmetries. All the vectors associated to those translations have the same direction and they are integer multiples of a determined non-zero vector $\vec{u}$.

Note: The remaining symmetries of a frieze can be the rotations of a 180º angle (half turn), reflexions or glide reflexions in relation to a parallel line to $\vec{u}$.

There are only seven distinct ways of repeating a motif over a frieze, resorting to the four types of symmetry (translation symmetry, rotation symmetry [of a 180º angle], reflexion symmetry, and glide reflexion symmetry).

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The identification of the group to which the frieze belongs can be made through some questions. Figure 3 demonstrates an algorithm used to determine the group of symmetry of the friezes regarding the 7 groups of friezes.

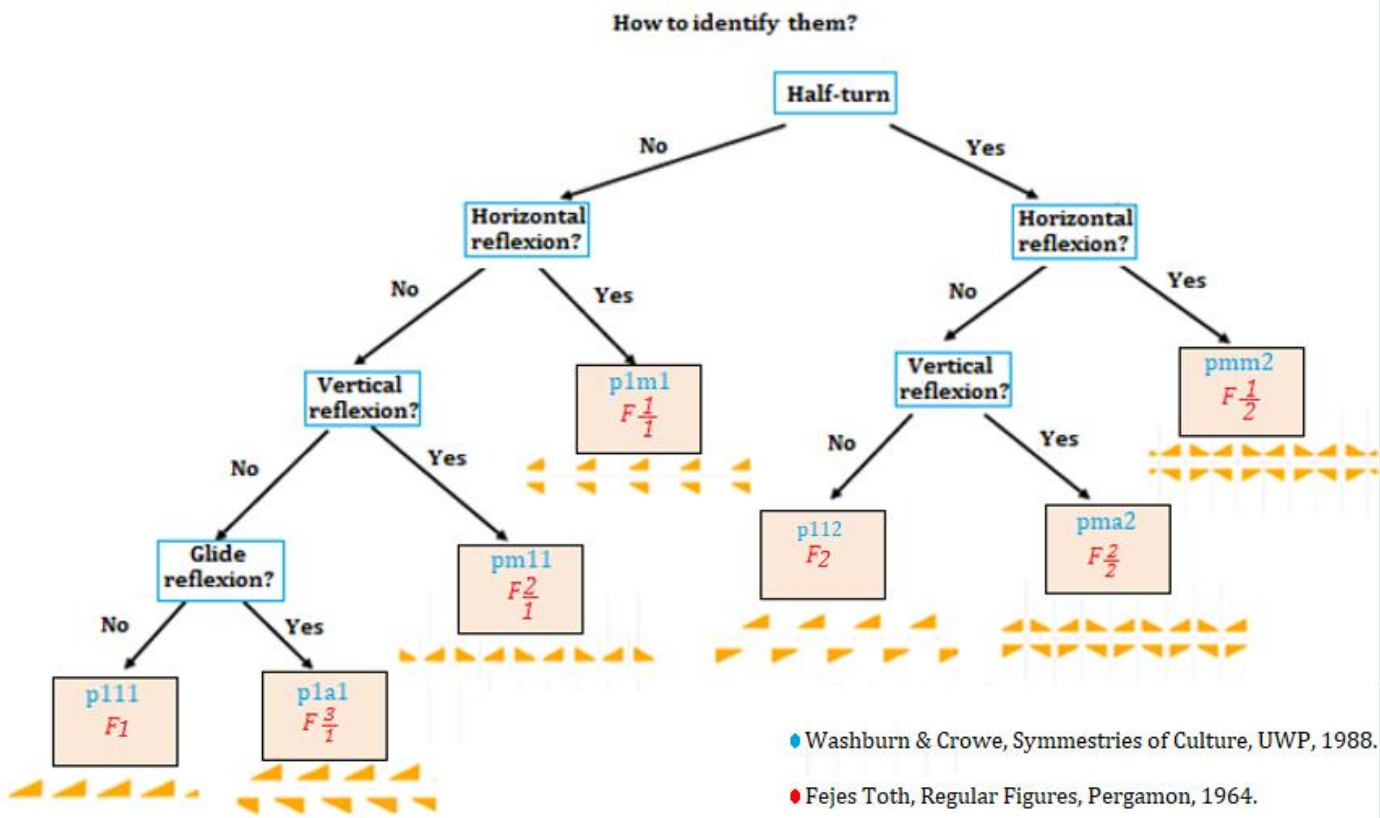

Fig. 3 – The seven types of friezes
(Source: Teixeira, R. C. (2013). Os sete grupos de frisos. Retrieved from: http://sites.uac.pt/rteixeira/files/2013/07/Fluxograma_Frisos_final.pdf (11.07.2019))

Pay attention to an example of each one of the 7 groups found in the pavements of the city of Angra do Heroísmo:

1. **Frieze only with translation**:

Fig. 4 – Frieze in pavement near Largo do Colégio
(Source: Teixeira, R. C. (2014, July 14). Os sete tipos de frisos em calçada de Angra do Heroísmo. Retrieved from: https://cienciapatodos.webnode.pt/news/os-sete-tipos-de-frisos-em-cal%C3%A7ada-de-angra-do-heroismo/ (11/07/2019))

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2. Frieze with glide reflexions and translation:

Frieze
p1a1
$F\frac{3}{1}$

Fig. 5 – Frieze in Rua da Queimada
(Source: Teixeira, R. C. (2013). Roteiro de Frisos da Cidade de Angra do Heroísmo. Retrieved from: https://www.mat.uc.pt/mpt2013/files/Roteiro-de-frisos-Angra.pdf (11.07.2019))

3. Frieze with vertical reflexions and translation:

Frieze
pm11
$F\frac{2}{1}$

Fig. 6 – Frieze in Rua de Cima de Santa Luzia
(Source: Teixeira, R. C. (2013). Roteiro de Frisos da Cidade de Angra do Heroísmo. Retrieved from: https://www.mat.uc.pt/mpt2013/files/Roteiro-de-frisos-Angra.pdf (11.07.2019))

4. Frieze with horizontal reflexions and translation:

Frieze
p1m1
$F\frac{1}{1}$

Fig. 7 – Frieze in pavement near Largo do Colégio
(Source: Teixeira, R. C. (2014, July 14). Os sete tipos de frisos em calçada de Angra do Heroísmo. Retrieved from: https://cienciapatodos.webnode.pt/news/os-sete-tipos-de-frisos-em-cal%C3%A7ada-de-angra-do-heroismo/ (11/07/2019))

5. Frieze with a rotation of 180º (half turn) and translation:

Frieze
p112
F_2

Fig. 8 – Frieze in Rua de São João
(Source: Teixeira, R. C. (2013). Roteiro de Frisos da Cidade de Angra do Heroísmo. Retrieved from: https://www.mat.uc.pt/mpt2013/files/Roteiro-de-frisos-Angra.pdf (11.07.2019))

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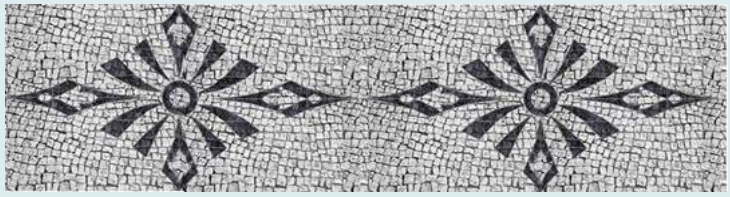
6. **Frieze with a rotation of 180º (half turn), a vertical reflexion, a glide reflexion, and translation**:

Frieze
pma2
$F\frac{2}{2}$

Fig. 9 – Frieze in Avenida Tenente Coronel José Agostinho
(Source: Teixeira, R. C. (2013). Roteiro de Frisos da Cidade de Angra do Heroísmo. Retrieved from: https://www.mat.uc.pt/mpt2013/files/Roteiro-de-frisos-Angra.pdf (11.07.2019))

7. **Frieze with a rotation of 180º (half turn), a vertical reflexion, a horizontal reflexion, and translation**:



Frieze
pmm2
$F\frac{1}{2}$

Fig. 10 – Frieze in Rua da Conceição
(Source: Teixeira, R. C. (2014, July 14). Os sete tipos de frisos em calçada de Angra do Heroísmo. Retrieved from: https://cienciapatodos.webnode.pt/news/os-sete-tipos-de-frisos-em-cal%C3%A7ada-de-angra-do-heroismo/ (11/07/2019))

These contents allowed to understand why Angra do Heroísmo is called the "City of the seven friezes". This recognition prizes its heritage regarding the pavements and allows to stimulate mathematical tourism.

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TASKS

TASK 1

In the figure below, a frieze is represented.

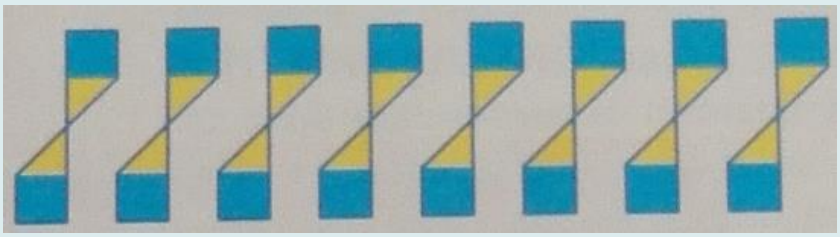

Fig. 11 – Frieze
(Source: Costa, B., & Rodrigues, E. (2015). Novo espaço 10: matemática A, 10º ano. Porto: Porto Editora)

1.1. Identify the isometry that applies to motif I in order to obtain motif II.

Fig. 12 – Motifs of the frieze
(Source: Costa, B., & Rodrigues, E. (2015). Novo espaço 10: matemática A, 10º ano. Porto: Porto Editora)

1.2. Which of the vectors represented in the figure below matches to translations that make the frieze invariant?

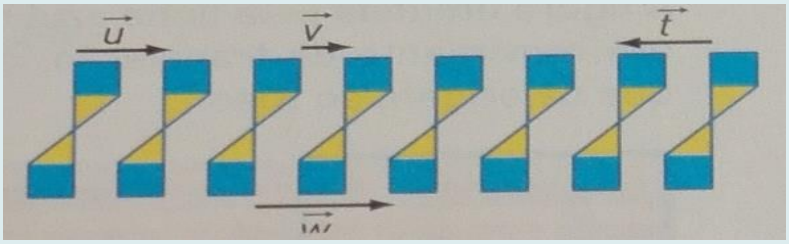

Fig. 13 – Frieze
(Source: Costa, B., & Rodrigues, E. (2015). Novo espaço 10: matemática A, 10º ano. Porto: Porto Editora)

TASK 2

Identify the symmetries that are present in each one of the following friezes:

2.1.

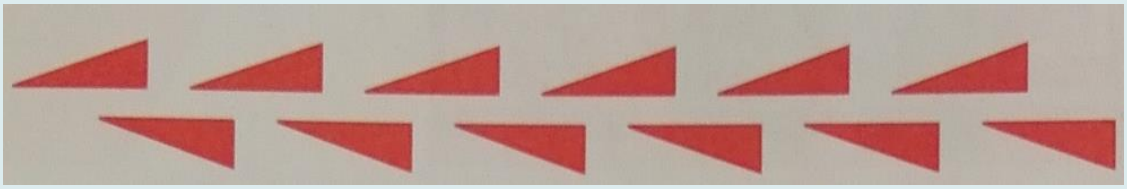

Fig. 14 – Frieze
(Source: Costa, B., & Rodrigues, E. (2015). Novo espaço 10: matemática A, 10º ano. Porto: Porto Editora)

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2.2.

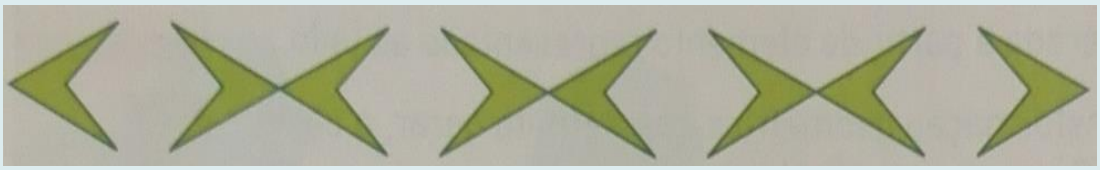

Fig. 15 – Frieze
(Source: Costa, B., & Rodrigues, E. (2015). Novo espaço 10: matemática A, 10º ano. Porto: Porto Editora)

2.3.

Fig. 16 – Frieze in Rua Dr. Henrique Braz
(Source: Teixeira, R. C. (2013). Roteiro de Frisos da Cidade de Angra do Heroísmo. Retrieved from: https://www.mat.uc.pt/mpt2013/files/Roteiro-de-frisos-Angra.pdf (11.07.2019))

TASK 3

Use the motif below to elaborate a frieze that has the symmetry of a horizontal reflexion.

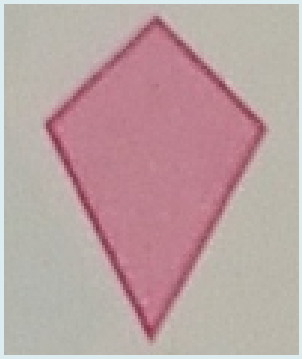

Fig. 17 – Motif of the frieze
(Source: Costa, B., & Rodrigues, E. (2015). Novo espaço 10: matemática A, 10º ano. Porto: Porto Editora)

TASK 4

Identify the isometries that make the frieze invariant for each one of the four friezes that are represented below.

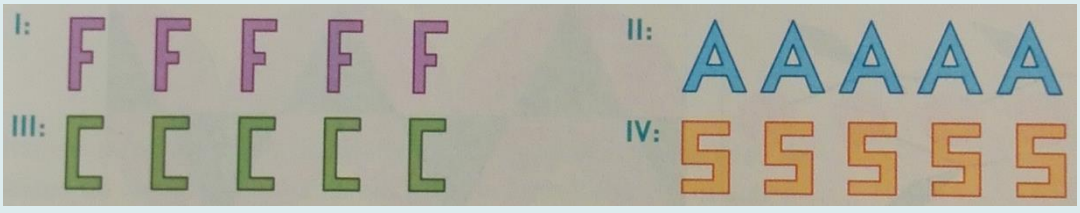

Fig. 18 – Friezes
(Source: Costa, B., & Rodrigues, E. (2015). Novo espaço 10: matemática A, 10º ano. Porto: Porto Editora)

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LEARN MORE...

The Origins of the Portuguese Pavement and its first exemplars

https://getlisbon.com/discovering/origins-of-the-portuguese-pavement/

Transformations, Congruence, and Similarity

https://www.khanacademy.org/math/basic-geo/basic-geo-transformations-congruence

Frieze Patterns

https://www.youtube.com/watch?list=PLkTaxm2IKUGgphZsrNr3PQTRsf7t7Gm_k&v=2bAbueWC0Q8 and https://www.youtube.com/watch?v=524gLKdaMZM

The mathematics of friezes explained

http://www.mathematicsinthemaking.eu/fileadmin/media/Output/Material_for_activities/14_12_08The_mathematics_of_friezes_explained_logos.pdf

Explore the Symmetries with GeCla – Generator and Classifier Programme

http://www.atractor.pt/mat/GeCla/index-_en.html

Explore the Isometries and Patterns with Geogebra

https://www.geogebra.org/classic

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