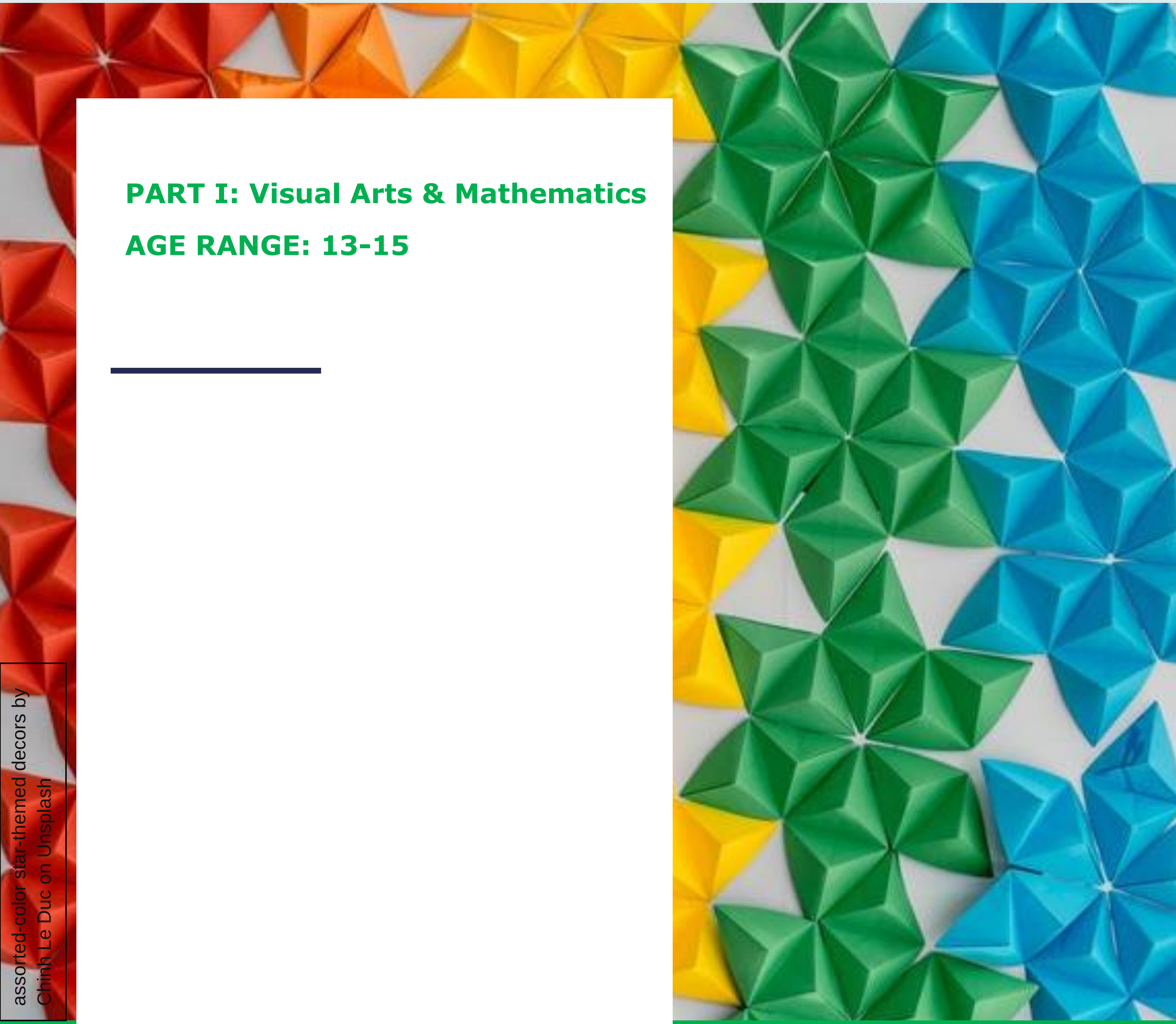

assorted-color star-themed decors by Chinh Le Duc on Unsplash

PART I: Visual Arts & Mathematics

AGE RANGE: 13-15

TOOL 5: ORIGAMI AND SPATIAL RELATIONS

LogoPsyCom

TheArtOfMaths

Co-funded by the Erasmus+ Programme of the European Union

Educator's Guide

Title: Origami and spatial relations

Age Range: 13-15 years old

Duration: 2 hours

Mathematical Concepts: dimensions in space, symmetry, geometric relations, Thales theorem, Pythagorean Theorem, Axioms

Artistic Concepts: Zhezhi, Origami, paper folding techniques, pop-up books

General Objectives: To discover how origami can illustrate mathematical concepts and famous theorems and acquire a more practical view of the use of mathematics.

Instructions and Methodologies: The students will explore both fields as a whole, by applying origami techniques. This is a basis to discover the mentioned concepts.

Resources: This tool provides pictures and videos for you to use. The topics addressed will help you find other materials to personalize and give nuance to your lesson.

Tips for the educator: Learning by doing is very efficient, especially with young learners with learning difficulties. Always explain what each math concept is useful for, practically. The origami activities can be done in pairs, especially for dyspraxic students who would have more difficulties with the manipulations.

Desirable Outcomes and Competences: At the end of this tool, the student will be able to:

- Know what Euclid's axioms were;
- Understand and use Thales' Theorem;
- Understand and use the Pythagorean Theorem.

Debriefing and Evaluation:

Write 3 aspects you liked about this activity:	1. 2. 3.
Write 2 aspects that you have learned	1. 2.
Write 1 aspect for improvement	1.

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Introduction

Origami is a Japanese word that refers to paper folding. This technique is clearly linked to mathematics as it makes use of spatial relations to create shapes which can be transformed by folding and unfolding the paper in specific ways.

Geometric knowledge could decisively be conceived as a theoretical instrument in visual arts. Our every manipulation in the three-dimensional space is a use of mathematics. Without even thinking about it, we are calculating distances and identifying special relations.

Many scholars have emphasized the value of using origami in education, especially to teach geometry. The paper folding techniques can help students understand geometric relations and transformations by experiencing and analyzing the changes they observe thanks to this creative method. The added value is that they can use the learnt concepts to build new artistic compositions and have a material result to their operations.

This tool will therefore focus on the applicability of mathematics in the folding techniques of origami and will include some physical manipulations for students to have a hands-on creative experience with math.

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The art of origami

Before it became part of Japanese art, paper folding appeared in China and was called Zhezhi. It is not until the 6th century that Buddhist monks brought this art to Japan. In Japanese, "origami" comes from "ori", fold, and "gami", paper. It was used as a leisure activity for children until a geometry teacher, Akira Yoshizawa, who had himself enjoyed origami as a child, decided to use it to teach angles, lines and shapes to his students. He developed his new techniques and what used to be a hobby became a form of art in which many math teachers found great interest.

Where can we see and use origami?

Origami can be used for many different purposes. Some math teachers use it to teach geometry but you can apply origami elsewhere too! You can even observe it in nature!

Did you know that some tree's leaves sometimes unfold in a very similar way to an origami technique called **Miura folding**? Well, it turns out that researchers at the Centre for Biomimetics, at the University of Reading found that the beech tree's and hornbeam tree's leaves unfold very similarly to the origami technique.

Figure 1: Beech Tree's leaves unfolding

Amongst the human uses of origami, Miura folding is also used with maps. They are folded in a way that makes them easy to transport and unfold.

Here is a GIF that shows what it looks like:
https://commons.wikimedia.org/wiki/File:Miura-ori.gif

This technique is very useful when you need to transport a large piece of paper or plane surface. It was therefore also used in the design of solar panels so that they would unfold more easily when placed at their final destination.

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Here is a video by the BYU (Brigham Young University) that explains how origami is used in the design of solar panels: https://www.youtube.com/watch?v=3E12uju1vgQ.

Pop-up Books

You probably know what pop-up books are. They contain folded figures that unfold when you open the book and show the illustration of the story. These books use paper folding in a very creative way.

The following GIF will show you what a pop-up book is: https://en.wikipedia.org/wiki/File:PopupCinderella.gif

You can watch this TED-Ed video to understand how to create animations with pop-up books: https://www.youtube.com/watch?v=RZR_b753ZJ0

Glossary

Zhezhi: the art of paper folding in China.

Origami: the art of paper folding in Japan.

Miura Folding is the use of lines, shapes, forms and colors that differ from the accurate depiction of the real world in visual art.

Pop-up books: are books in which the authors use folded paper to create 3D illustrations that unfold as the reader opens the book.

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The math behind origami

The Axioms:

Origami can be complementary to the geometry rules we already know. For instance, the axioms in Euclid's "Elements" can be a good example to create another set of axioms related to the art of paper folding.

Let's first see what **Euclid's axioms** were:

- Given any two points, one can draw a straight line between them;
- Any line segment can be extended indefinitely;
- Given a point and a line segment starting at the point, one can describe a circle with the given point as its centre and the given line segment as its radius;
- All right angles are equal to each other;
- Given a line and a point P that is not on the line, there is one and only one line through P that never meets the original line.

Although these axioms can help demonstrate more complex theorems, origami geometry can also be very valuable.

Here are the **Huzita–Hatori axioms**:

The **fold** and the **movement** are highlighted in on each image.

1. Given two points **P1** and **P2**, there is a unique fold that passes through both of them.

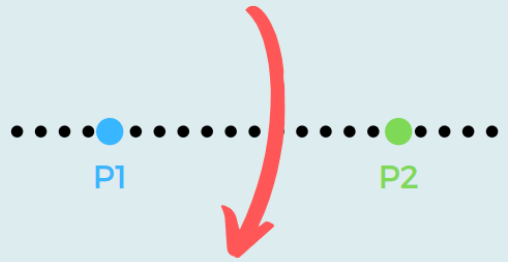

Figure 2: First Huzita-Hatori Axiom

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2. Given two points **P1** and **P2**, there is a unique fold that places P1 onto P2.

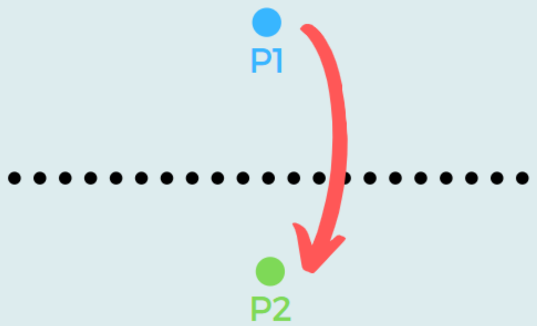

Figure 3: Second Huzita-Hatori Axiom

3. Given two lines **L1** and **L2**, there is a fold that places L1 onto L2.

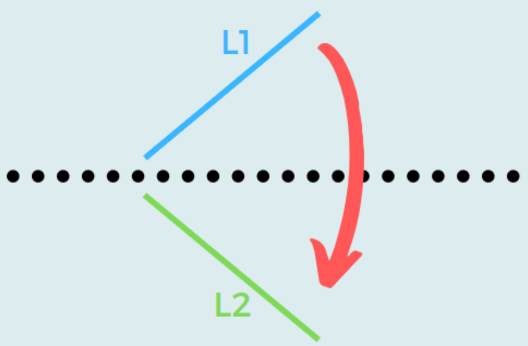

Figure 4: Third Huzita-Hatori Axiom

4. Given a point **P1** and a line **L1**, there is a unique fold perpendicular to L1 that passes through P1

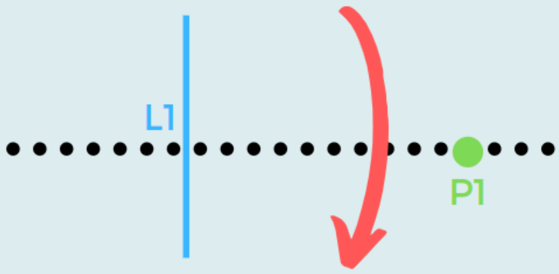

Figure 5: Fourth Huzita-Hatori Axiom

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5. Given two points **P1** and **P2** and a line **L1**, there is a fold that places P1 onto L1 and passes through P2.

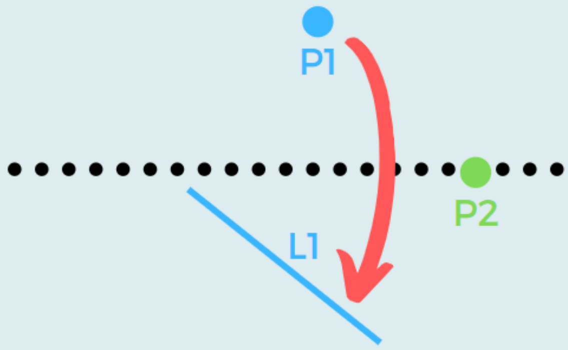

Figure 6: Fifth Huzita-Hatori Axiom

6. Given two points **P1** and **P2** and two lines **L1** and **L2**, there is a fold that places P1 onto L1 and P2 onto L2.

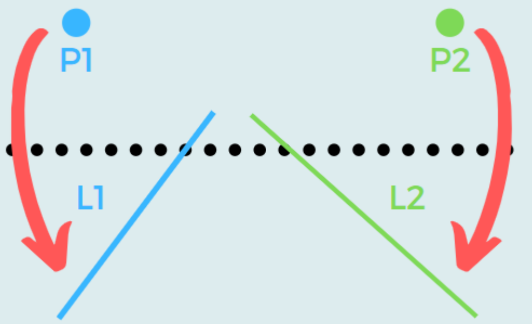

Figure 7: Sixth Huzita-Hatori Axiom

7. Given one point **P1** and two lines **L1** and **L2**, there is a fold that places P1 onto L1 and is perpendicular to L2.

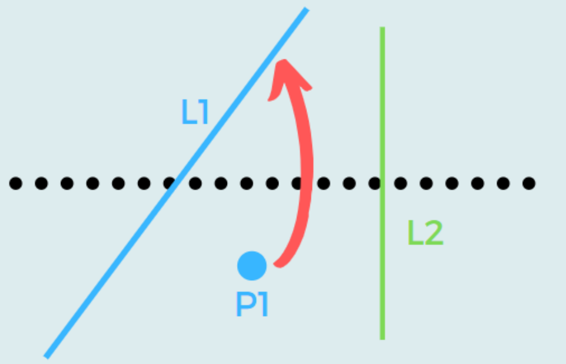

Figure 8: Seventh Huzita-Hatori Axiom

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Thales' Theorem

Thales was a Greek mathematician from the Antiquity. His theorem is mentioned in Euclid's "Elements".

if A, B, and C are distinct points on a circle where the line AC is a diameter, then the angle ∠ABC is a right angle.

To demonstrate it, Thales explained it as follows:

- Since OA = OB = OC, the triangles OBA and OBC are isosceles triangles,

and by the equality of the base angles of an isosceles triangle,

- ∠OBC = ∠OCB and ∠OBA = ∠OAB.

Let **a = ∠BAO** and **β = ∠OBC**.

- The three internal angles of the ABC triangle are a, (a + β), and β.
- Since the sum of the angles of a triangle is equal to 180°, we have:

$a + (a + \beta) + \beta = 180°$

$2a + 2\beta = 180°$

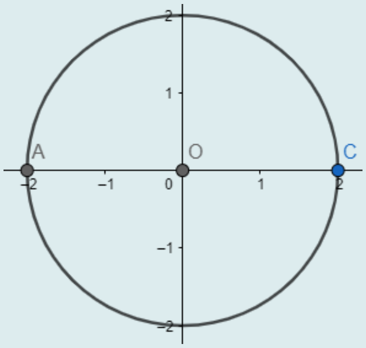
$2(a + \beta) = 180°$

$a + \beta = 90°$

Q.E.D.

Let's draw the geometrical proof of the theorem:

1. With a ruler and compass, draw a circle on a sheet of paper.



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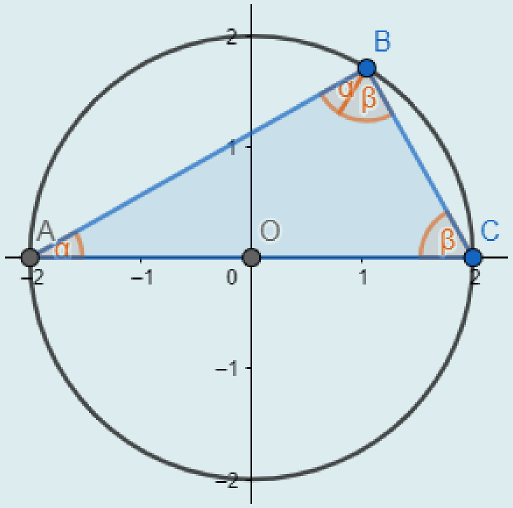
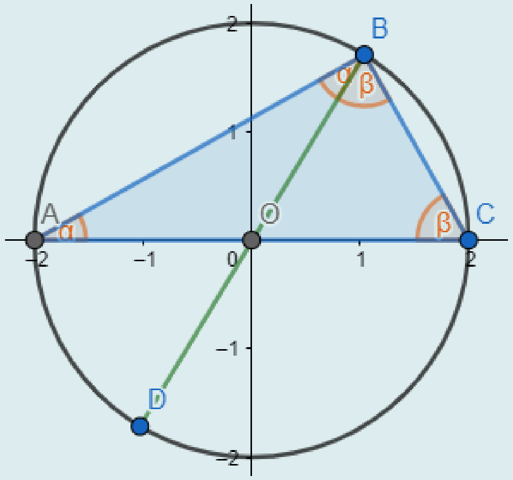
Erasmus+

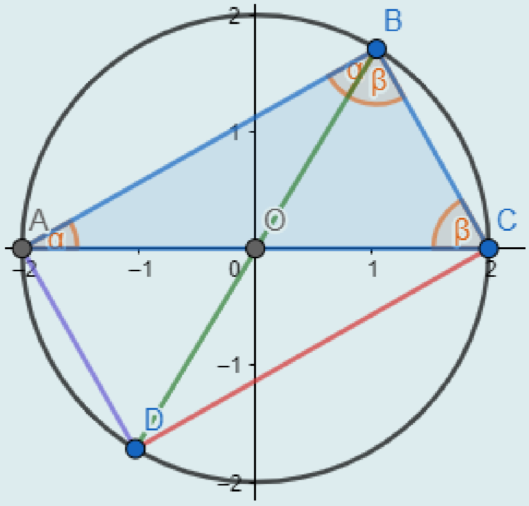
2. Draw a triangle ABC where the **segment AC** is the **diameter** of the circle;



3. **From point B**, draw a **segment** that goes **straight through the center O** and ends at an **intersection D** of the circle;



4. You are now able to draw a parallelogram with all points (ABCD).



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a) Is this parallelogram a rectangle?

____

b) Are all its angles right angles?

____

c) What does it mean?

____

d) How do you think we can use origami here?

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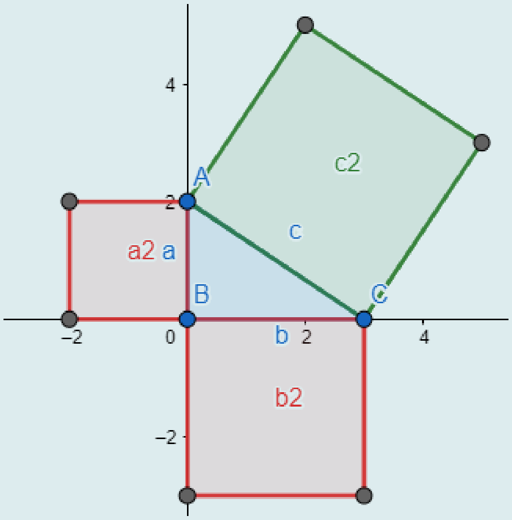
Pythagorean Theorem

Pythagoras was also a Greek philosopher and mathematician from the Classical Antiquity. He is most famous for his theorem which says that:

the square of the **hypotenuse** (the side **opposite** the right angle) is equal to the sum of the squares of the other two sides.
We write it: $a^2+b^2 = c^2$



In this example:

- a = segment AB
- b = segment BC
- c = segment CA

➔ The red squares represent a^2 and b^2

➔ The green square represents c^2

➔ The segment AC is the **hypotenuse**

Let's do some exercises using the formula:

For each triangle:

1. Name the segments a, b and c on the picture;
2. Use the Pythagorean equation;
3. Calculate the length of the hypothenuse.

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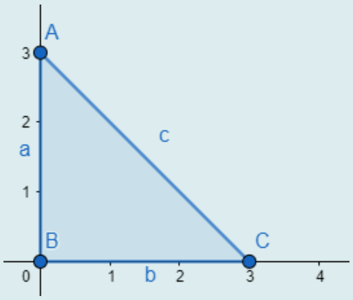

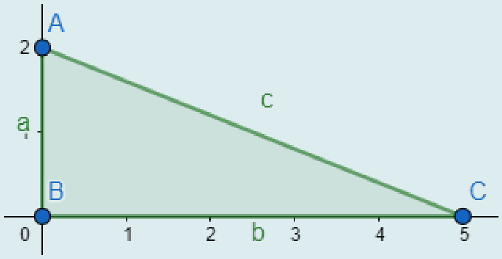

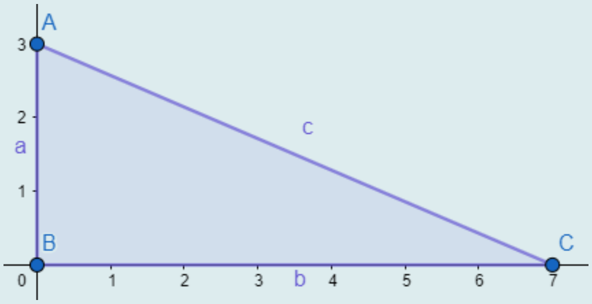

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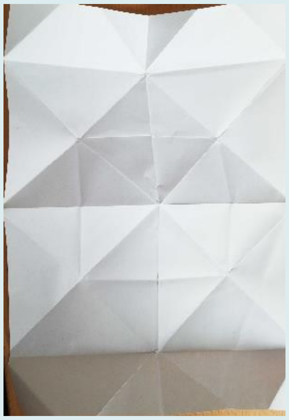
TASK

This task will enable you to understand the ways in which origami can represent math concepts and techniques.

Drawing the Pythagorean Theorem with an origami technique: the boat!

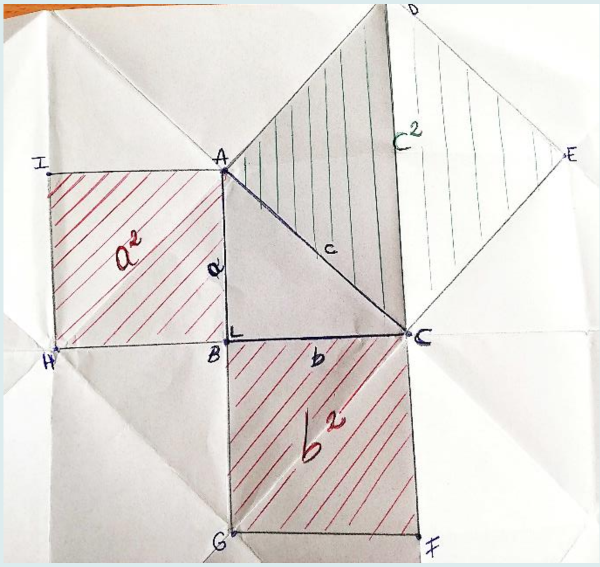
Watch the following video to learn how to do it: https://www.youtube.com/watch?v=Cjx3My0kDtY&feature=youtu.be

Now that you have your boat, unfold it by doing everything backwards.
You should have something like this:



You can see that there are some triangles and squares everywhere on the page.

Let's draw the Pythagorean Theorem on the created pattern!



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1. Answer the following questions to see if some origami axioms are followed:

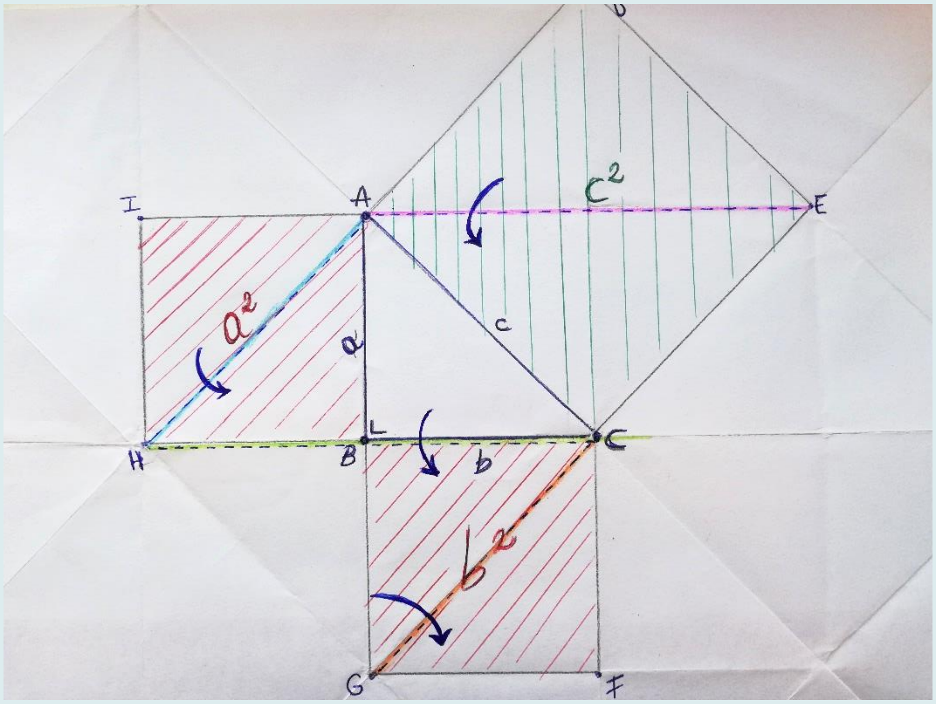
a. Is there a fold that passes through the points A and E?

b. Is there a fold with which point I can be put onto point B?

c. Is there a fold that places the segment GB onto segment GF?

d. Is there a fold perpendicular to segment AG that passes through point C?

2. Highlight those folds in a different color on your drawing.



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LEARN MORE…

Paper on the use of origami in school:

http://www.fau.edu/education/centersandprograms/mathitudes/documents/20080901bMathitudesOct08revisionFinalVersionforpublicationOct242008.pdf

TED Talk on the contribution of mathematics to the art of origami:

https://www.ted.com/talks/robert_lang_folds_way_new_origami#t-193336

TED-Ed video to go further with the Pythagorean Theorem:

https://www.youtube.com/watch?v=YompsDlEdtc

Article on the math in origami:

https://theconversation.com/origami-mathematics-in-creasing-33968

Article on the origami history and axioms:

https://plus.maths.org/content/power-origami

Article on the math in origami:

https://www.tor.com/2017/06/29/the-magic-and-mathematics-of-paper-folding/

TED-Ed video about pop-up books:

https://www.youtube.com/watch?v=RZR_b753ZJ0

How to fold a boat in origami (images + video):

http://www.origami-instructions.com/origami-boat.html

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