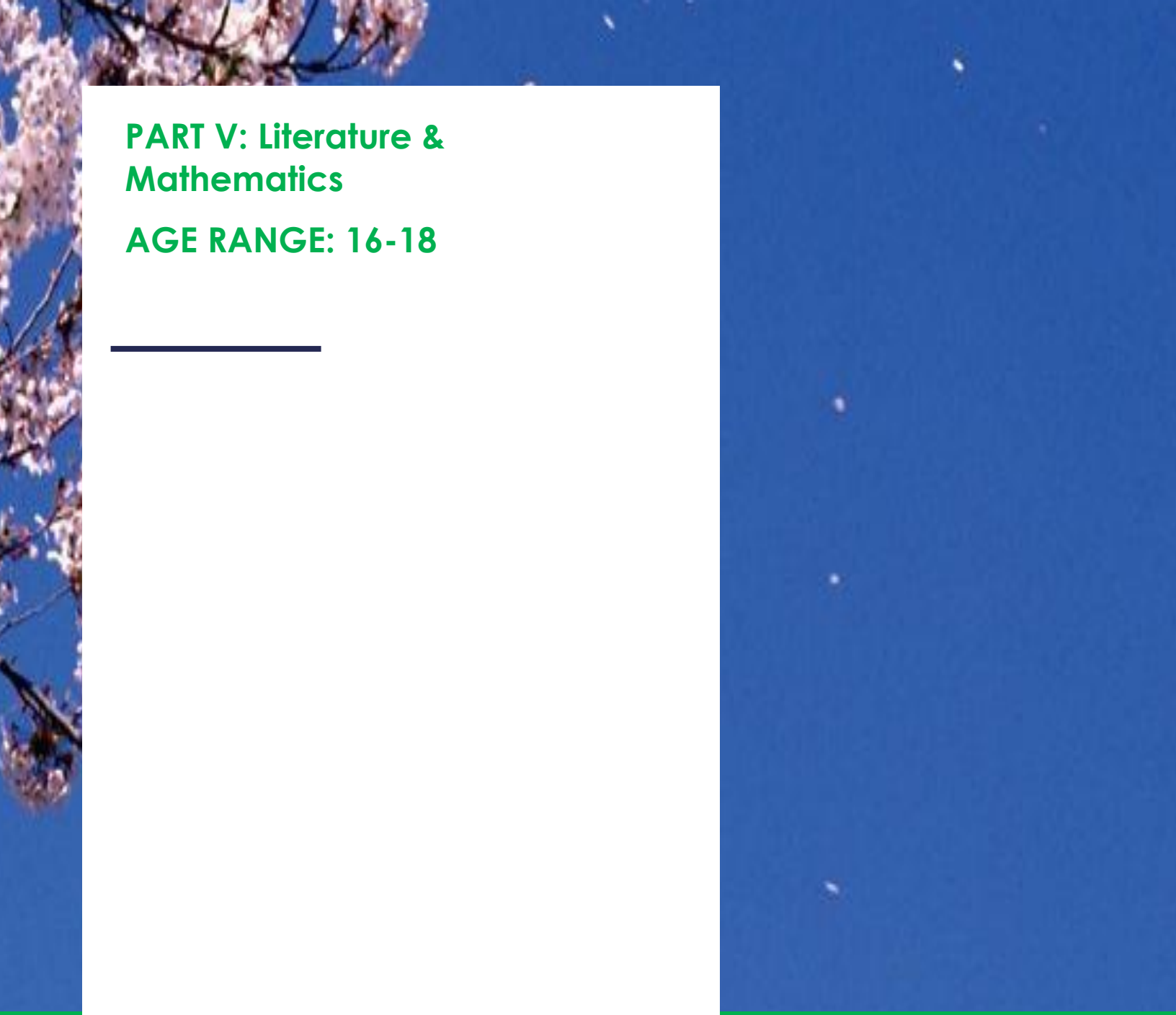

PART V: Literature & Mathematics

AGE RANGE: 16-18

TOOL 42: APPROACHING TRIANGULAR NUMBERS THROUGH THE BOOK "THE HOUSEKEEPER AND THE PROFESSOR"

C.I.P. Citizens In Power

Co-funded by the Erasmus+ Programme of the European Union

Educator's Guide

Title: Approaching Triangular Numbers through the book 'The housekeeper and the Professor' by Yoko Ogawa (2009)

Age Range: 16-18 years old

Duration: 1.5 - 2 hours

Mathematical Concepts: Triangular Numbers

Artistic Concepts: Literature and Mathematics

General Objectives: The students will firstly identify -through the introductory part- the connection and benefits of learning mathematics through literacy documents. They will then identify one of the most important books in this field, get acquainted with the author and the book overview. Closer to the end, they will have the chance to read one small part deriving from the Chapter 4 of the Book 'The housekeeper and the Professor' and analyze some of its mathematical concepts through the mathematical task. This is to reach the ultimate mathematical goal which is to interact with the Triangular Numbers.

Instructions and Methodologies: It is preferable to follow the structure of this tool as it begins with some simple background information on the connection between literacy and mathematics in general, whilst getting into more details and its benefits progressively. A biography of the author and an overview of the book are given before reaching to the actual passage of Chapter 4 of the Book, related to the Triangular Numbers and the associated Mathematical task.

Resources: This tool provides an overview of the book, pictures of both the author and the cover of the book, a youtube trailer on the film 'The Professor's Beloved Equation' which is based on the book 'The housekeeper and the Professor'; and the main Mathematical task.

Tips for the educator: It will be important to grasp the interest of your students through giving emphasis on how literature is associated with mathematics and the many benefits it will have, as well as the actual fragment of the text given, which will help the students to see and comprehend on the reasoning part of a complicated

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formula, related to triangular numbers, in order to fully understand it and ultimately apply it to the mathematical task given.

Desirable Outcomes and Competences: Students will

- familiarize with the triangular numbers, as those are shown in the texts,
- apply the formula through simple steps based on the passage of Chapter 4 of the Book and
- solve the mathematical task.

Debriefing and Evaluation Questions: You can use these cards sometimes called EXIT CARDS either by a hard made from before or simply by posing these statements on board and the students can write their answers on a paper which they will leave preferably anonymously while exiting the room. The specific formative strategy is called 3,2,1. For more strategies you can visit: https://www.bhamcityschools.org/cms/lib/AL01001646/Centricity/Domain/131/70%20Formative%20Assessments.pdf

3-2-1	
Write 3 things you liked about this activity	**1.** **2.** **3.**
Write 2 things you have learned	**1.** **2.**
Write 1 aspect for improvement	**1.**

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Introduction

According to Cohen (2013) "Studying mathematics-related fiction and poetry helps students develop an appreciation for both mathematics and literature and an understanding of the connection between the two". There are many studies coming from researchers such as from Growney (2008, 2009), Bahls (2009), Glaz and Liang (2009), Glaz (2010, 2011), and Ivy (2004, 2009) explaining how mathematics can be combined with literature in the classrooms. Generally connecting mathematics with arts, such as cinematography, drama, and language arts, has been considered as a helpful strategy for instructing mathematics for several reasons. Research has proven that providing an environment that is considered to be less stressful and psychologically safe increases students' inspiration and results (Jensen, 1998). When students are worried, achievement lessens because they are preoccupied with a nervous and worrying feeling, distracting them from the actual mathematical tasks (Covington 1999). These worried students could miss a lot of the information they are intended to learn because their focus is derived and deprived from their fear of mathematics, instead of the task to be learnt (Siegel 1999). Using literature effectively with mathematics can help reduce the anxiety felt by 'mathephobes' (Zambo, 2005).

Also, a very broadly cited article by Furner, Yahya and Duffy (2005), suggesting 20 strategies to teach mathematics reaching all students, includes in one of them, the use of literature emphasizing these benefits:

- "Teaches math concepts in the context of a story
- Incorporates integrated studies with reading, writing, speaking, listening, and so on
- Develops mathematical thinking
- Prevents math anxiety and creates a less math-anxious classroom environment
- Allows for a variety of responses
- Makes historical, cultural, and practical application connections
- Allows for the use of manipulatives as it relates to the story

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- Assesses a child's understanding by reading/questioning
- Offers a wide range of books that can be used to teach most math concepts
- Leads to problem solving and active involvement from the story's context
- Provides a shared experience for students and teacher" (Furner et al, 2005, p. 22)

There are many literacy books in several languages associated with mathematics. Giving a small number of the most known ones are 'The Devotion of suspect X' by Keigo Higashino (1958), 'The calculus wars' by Jason Socrates Bardi (2006), 'Logicomix' by Apostolos Doxiades (2009), 'Mathematical Mysteries: The Beauty and Magic of Numbers' by Calvin Clawson (1999) and the one we have chosen for this task which is 'The housekeeper and the Professor' by Yoko Ogawa (2009) a bestselling Japanese novel also translated in English.

'The housekeeper and the Professor' is composed of 11 chapters, dealing with several mathematical concepts, such as square roots, amicable numbers, the number theory, prime numbers, Gauss' formula and Mersenne primes. Here, an overview of the book will be given and only one mathematical task related to Chapter 4 and the Triangular numbers. 5

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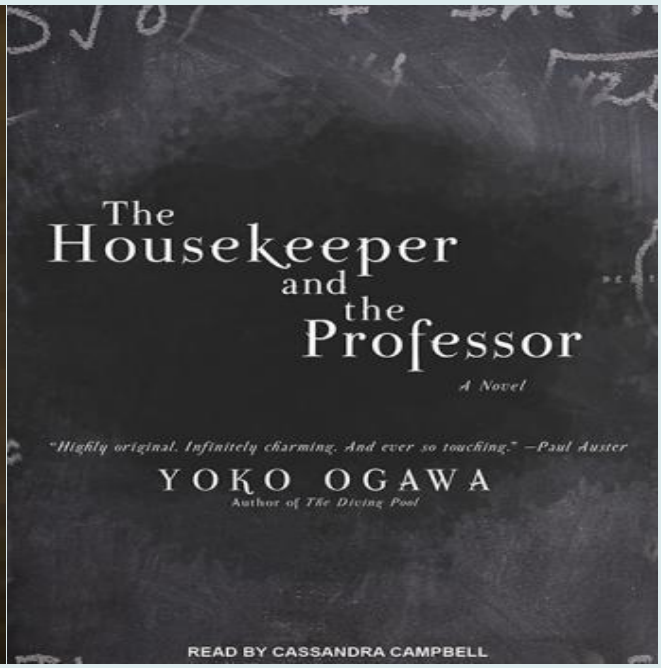
Biography

Picture 1: Yoko Ogawa (retrieved from: https://www.google.com/search?q=yoko+ogawa&client=firefox-b-d&source=lnms&tbm=isch&sa=X&ved=0ahUKEwiMsdjB_ODiAhXCEVAKHX7WCOIQ_AUIECgB&biw=1138&bih=527#imgrc=l3mrvd3faKaIvM)



Picture 2: Cover page of 'The housekeeper and the professor' https://www.google.com/search?q=the+housekeeper+and+the+professor&client=firefox-b-d&source=lnms&tbm=isch&sa=X&ved=0ahUKEwj41ev9_ODiAhUKJFAKHTTpDfkQ_AUIECgB&biw=1138&bih=527#imgrc=cPXLj_9wuxL-GM)

Yoko Ogawa was born in Okayama, Okayama Prefecture, graduated from Waseda University, and lives in Ashiya, Hyōgo, with her husband and son. Since 1988, Ogawa has published more than forty works of fiction and nonfiction. In 2006 she co-authored "An Introduction to the World's Most Elegant Mathematics" with Masahiko Fujiwara, a mathematician, as a dialogue on the extraordinary beauty of numbers. Kenzaburō Ōe has said, "Yoko Ogawa is able to give expression to the most subtle workings of human psychology in prose that is gentle yet penetrating." The subtlety in part lies in the fact that Ogawa's characters often seem not to know why they are doing what they are doing. She works by accumulation of detail, a technique that is perhaps more successful in her shorter works; the slow pace of development in the longer

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works requires something of a deus ex machina to end them. The reader is presented with an acute description of what the protagonists, usually female, observe and feel and their somewhat alienated self-observations, some of which reflect Japanese society and especially women's roles within it. The tone of her works varies, across the works and sometimes within the longer works, from the surreal, through the grotesque and the —sometimes grotesquely— humorous, to the psychologically ambiguous and even disturbing. (Hotel Iris, one of her longer works, is more explicit sexually than her other works and is also her most widely translated.)

Original source from Wikipedia (Retrieved from: https://en.wikipedia.org/wiki/Y%C5%8Dko_Ogawa)

Overview of the book 'The housekeeper and the professor'

This book is about a 64 year-old mathematician. He used to be a professor 17 years before, but a car accident caused him a brain damage, and more specifically the part of the brain connected to memory. Now he only has a memory for 80 minutes, though he remembers facts that happened before the accident. His research field is number theory, but he is no more active in re-search because of the damage in his brain. Due to his memory problem, it is rather impossible to have a normal life. Everything seems to him to occur rapidly. This problem also caused him to give up from society and from relationships with other people. The professor lives with his sister-in-law, who has been a widow for many years. She lives in the main house and he lives in a very small detached house. They come from a rich family and make a living from the real estate they hold. They don't have much contact, at least after the accident. The sister-in-law hires a housekeeper to look after the professor. The housekeeper comes every day from the morning and makes breakfast, does laundry, cleans, and cooks dinner. She is twenty-nine years old, unmarried but has a ten year old boy. In Japan, it is not so common to hire a housekeeper. It is more common in very rich families. It is also rather unusual for a woman to have a child without being married.

The housekeeper arrives to the professor's house for the first time in March, 1992. The book is about the relationship shared by the professor, the housekeeper, and her son.

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As a mathematics professor his love has always been numbers and it is the one thing he mostly cares about and to some extent baseball, though he does not really keep up with the results. Quite a few real stories about Japanese baseball are quoted in the book. When the housekeeper comes to the professor's house every morning, he does not remember her. A memory technique that he uses is to keep many small pieces of paper clipped to his jacket to remind him of necessary information. One of them is about the housekeeper. Every morning when she comes, he identifies her with the face on one of the pieces of paper, and then he asks of his regular questions: all related to numbers such as how old are you?, what is your shoe size? And so on. The questions are always related to numbers. For example, if she says her shoe size is 24, then he says that's 4 factorial. Then she asks what factorial is, and he replies, and that is a classical conversation between them.

When the professor discovers about his housekeeper's son he reacts outrageously and asks from her to bring him to the house. This is how the relationship between the three people begins. The book does not involve any big drama. But every little thing in the professor's life is strange and hard for the people around him because of his memory problem. In addition to his memory problem, his leaning for numbers, especially prime numbers, is so strong that his life tends to be slightly comical or awkward. The professor tells many stories about math and numbers to the housekeeper and her son. One of their favorite things is the famous formula of Euler, $e^{\pi i} = -1$. They knew π but did not know i and e, so the professor thought they really didn't get the meaning of the formula.

 The Professor's Beloved Equation is a Japanese film released January 21, 2006 and directed by Takashi Koizumi. It is based on the novel The Housekeeper and the Professor. The trailer can be found on https://www.youtube.com/watch?v=pOV-jadkgAw

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Excerpt from Chapter 4

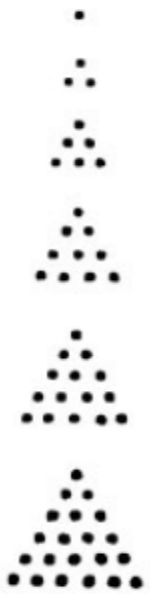
The clinic was old and depressing. The ceiling was discolored, and the grimy slippers stuck to your feet. Yellowed posters on the walls gave instructions for weaning and inoculations. The only light in the hall was the dim bulb outside the X-ray room.

They'd said the test was just a precaution, but Root had been in the examination room for some time.

"Have you ever heard of triangular numbers?" the Professor said, pointing at the radiation sign on the door of the X-ray room. It was shaped like a triangle.

"No," I said. He sounded calm now, but I could tell that he was still a little shaken.

"They're truly elegant," he said, beginning to draw dots on the back of a questionnaire that he'd picked up in the lobby.



"What do you make of these?"

"Well, let's see. It looks like neatly stacked firewood, or maybe rows of beans."

"That's right, the point is they're 'neatly' arranged. One in the first row, two in the second, three in the third.... It's the simplest way to form a triangle." I glanced at the dots on the page. The Professor's hand was trembling slightly. The black marks seemed to float up in

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the half-light. "So then, if we total up the number of dots in each triangle, we get 1, 3, 6, 10, 15, and 21. And if we write these as equations:

1

$1 + 2 = 3$

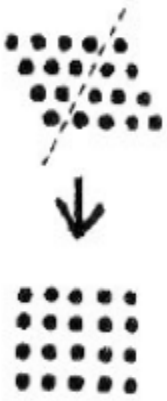
$1 + 2 + 3 = 6$

$1 + 2 + 3 + 4 = 10$

$1 + 2 + 3 + 4 + 5 = 15$

$1 + 2 + 3 + 4 + 5 + 6 = 21$

"In other words, a triangular number is the sum of all the natural numbers between 1 and a certain number. Then, if you put two of these triangles together, things get even more interesting. Why don't we look at the fourth one, 10, so we don't have to draw too many dots?"



It wasn't particularly cold in the hall, but the trembling in his hand had grown worse and the dots had slightly smudged. His whole

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being seemed concentrated in the tip of his pencil. A few of the notes on his suit were smeared with blood and now illegible.

"Look at this. When you put two of the four-row triangles together, you get a rectangle that is 4 dots high and 5 dots wide; and the total number in the rectangle is 4 x 5 or 20 dots. Do you see that? And if you divide that in half, you get 20 / 2 = 10, or the sum of the natural numbers from 1 to 4. Or, if you look at each line of the rectangle, you get:

$$\begin{array}{cccc} 1 & 2 & 3 & 4 \\ + & + & + & + \\ 4 & 3 & 2 & 1 \\ \hline 5 & 5 & 5 & 5 \end{array}$$

"And once you know that, you can use this relationship to figure out the tenth triangle--the sum of the numbers from 1 to 10--or the hundredth or any other. For 1 to 10 it would be:

$$\frac{10 \times 11}{2} = 55$$

"And for 1 to 100,

$$\frac{100 \times 101}{2} = 5050$$

"And 1 to 1000,

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$$\frac{1000 \times 1001}{2} = 500500$$

"And 1 to 10,000...."

The pencil rolled out of his hand and fell at his feet. The Professor was crying. I believe it was the first time I saw him in tears, but I had the feeling that I'd seen these emotions many times before. I placed my hand on his.

"Do you understand?" he said. "You can find the sums of all the natural numbers."

"I understand."

"Just by lining up the dots in a triangle. That's all there is to it."

"Yes, I see that now."

"But do you really understand?"

"Don't worry," I told him. "Everything's going to be all right. How can you cry, look at these beautiful triangular numbers."

Just then the door to the examination room opened and Root emerged.

"See!" he said, giving his bandaged hand a wave. "I'm fine."

Retrieved from: https://ibaracaldo.files.wordpress.com/2013/06/ogawa-yoko-the-housekeeper-and-the-professor.pdf

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The Math behind 'The housekeeper and the professor'

Glossary

Consecutive numbers: numbers that follow each other in order. For example: 1, 2, 3, 4, 5, …. or -100, -99, -98, -97, …. are sets of consecutive numbers.

Number theory: Number theory (or arithmetic or higher arithmetic in older usage) is a branch of pure mathematics devoted primarily to the study of the integers. German mathematician Carl Friedrich Gauss (1777–1855) said, "Mathematics is the queen of the sciences—and number theory is the queen of mathematics." Number theorists study prime numbers as well as the properties of objects made out of integers (for example, rational numbers) or defined as generalizations of the integers (for example, algebraic integers).

Prime numbers: A prime number is a whole number greater than 1 whose only factors are 1 and itself. A factor is a whole numbers that can be divided evenly into another number. The first few prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23 and 29. Numbers that have more than two factors are called composite numbers. The number 1 is neither prime nor composite.

Square number: a square number is the result of multiplying a number by itself. Squaring is the same as raising to the power 2, and is denoted by a superscript 2; for instance, the square of 3 may be written as 3^2, which is the number 9.

Triangular number: A triangular number or triangle number counts objects arranged in an equilateral triangle. The n^{th} triangular number is the number of dots in the triangular arrangement with n dots on a side, and is equal to the sum of the n natural numbers from 1 to n. The sequence of triangular numbers (sequence A000217 in the OEIS), starting at the 0th triangular number, is
0, 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105, 120, 136, 153, 171, 190, 210, 231, 253, 276, 300, 325, 351, 378, 406, 435, 465, 496, 528, 561, 595, 630, 666…

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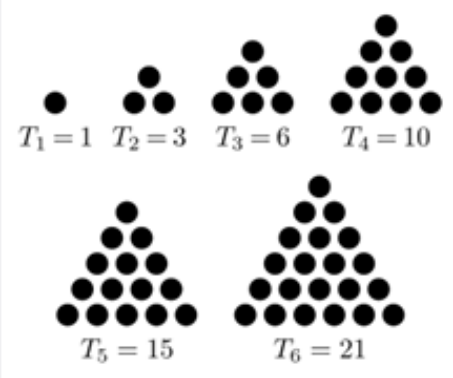
A "triangular number! or "triangle number" counts objects arranged in an equilateral triangle. The nth triangular number is the number of dots in the triangular arrangement with n dots on a side, and is equal to the sum of the n natural numbers from 1 to n.

The n^{th} triangular number is being expressed as T_n.

The first six triangular numbers are depicted below:



For instance, $T_3=6$ where n=3 is the size of the triangle's side, where 6 is the number of dots needed for the triangle to be depicted.
You can use an explicit formula in order to estimate a triangular number; the formula is given below:

$$T_n = \sum_{k=0}^{n} k = \frac{n(n+1)}{2}$$

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TASK

(a) Use the formula given above to estimate the triangular numbers T_4 and T_5

(b) Estimate the number of dots as well as the length of the triangular arrangement for T_4 and T_5

(c) Make the triangular arrangements for T_4 and T_5

(d) Estimate the sum T_4+T_5

(e) Proof that the sum of two consecutive triangular numbers is always a square number (perfect square)

Note: Refer to the Glossary for the definitions of: consecutive numbers/integers; square number.

(f) Do the results of (d) reflect to what had been proven in (e)?

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LEARN MORE…

If you want to further investigate on the topics addressed in this tool, you may go through the following links:

Books used in the introduction on math related to literature:

Cohen, M. D. (2013). *Truth & beauty: Mathematics in literature.* (No. 106). Mathematics Teacher. Retrieved from http://search.ebscohost.com/login.aspx?direct=true&AuthType=ip,sso&db=eric&AN=EJ1018180&site=eds-live&custid=s1098328
http://www.nctm.org/publications/article.aspx?id=35612

Furner, J. M., Yahya, N., & Duffy, M. L. (2005). Teach mathematics: Strat-egies to reach all students. Intervention in School and Clinic, 41, 16–23.

Zambo, R. (2005). The Power of Two: Linking Mathematics and Literature. *Mathematics Teaching in the Middle School, 10*(8), 394-399. Retrieved from http://www.jstor.org/stable/41182121

Book review:

https://www.ams.org/notices/201005/rtx100500635p.pdf?fbclid=IwAR1pCMwgeYrT42v-crjS43ttgGchjpaoOK3q4lGmrcNxyaxlNEqOWFEUMjY

Glossary, Triangular numbers:

https://en.wikipedia.org/wiki/Triangular_number?fbclid=IwAR03qFwv1dxaBAIXkdh_gAmGALDjdL5Evr448uJyAVAZueMYTHD-CAjoaLI

The Book:

https://ibaracaldo.files.wordpress.com/2013/06/ogawa-yoko-the-housekeeper-and-the-professor.pdf

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