PART I: VISUAL ARTS & MATHEMATICS

AGE RANGE: 16-18

TOOL 8: GOLDEN RATIO IN ARTS AND ARCHITECTURE

C.I.P. Citizens In Power

Co-funded by the Erasmus+ Programme of the European Union

Educator's Guide

Title: Golden Ratio in Arts and Architecture

Age Range: 16-18 years old

Duration: 2 hours

Mathematical Concepts: Golden Mean and Golden Ratio

Artistic Concepts: The Golden Ratio behind Arts and Architecture.

General Objectives: The students will discover the beauty of visual arts which have been created using geometry. They will familiarize with architectural masterpieces and paintings that have been made based on the golden ratio. Students will ultimately understand what the golden ratio is and use it to solve mathematical tasks.

Instructions and Methodologies: The teacher can start with the Youtube video: https://www.youtube.com/watch?v=6nSfJEDZ_WM until minute 1.41, before getting into more detail <the introduction, the most famous architectural designs and paintings> and then analyzing the mathematical aspects, such as the formula of golden ratio, through the 'Golden Ratio Explained' section. After the usage of interactive material along with the YouTube Videos, pictures and small exercises, students should be able to solve the actual mathematical tasks.

Resources: The tool is based mostly on pictures, YouTube Videos and academic resources, whilst it will be useful if the students can work on a computer to visit the sites suggested, and especially for trying the slider for Golden Ratio suggested as exercises in section 'Golden ratio explained'.

Tips for the educator: The Golden Ratio is so universal that it would be hard **not** to capture the attention of your audience. For this though, the introduction would help to understand better what the golden ratio is about, and where it can be found through pictures and interesting videos. Then there are some very useful and 'easy' tasks in section 'Golden ratio explained', before getting into more advanced mathematical exercises, relating Visual Arts and Mathematics through the Mathematical Tasks exercises.

This project has been funded with support from the European Commission. This publication reflects the views only of the author, and the Commission cannot be held responsible for any use which may be made of the information contained therein.

Desirable Outcomes and Competences:

- Make students interested in geometric shapes and constructions
- Relate geometry to features we see daily (on our body and nature), but also in masterpieces of Visual Arts.
- Be competent in using Golden Ratio in mathematical tasks and finding it even when 'hidden' in art pieces.

Debriefing and Evaluation Questions: The specific formative strategy is called 3,2,1. For more strategies you can visit:

https://www.bhamcityschools.org/cms/lib/AL01001646/Centricity/Domain/131/70%20Formative%20Assessments.pdf

3-2-1	
Write 3 things you liked about this activity	1. 2. 3.
Write 2 things you have learned	1. 2.
Write 1 aspect for improvement	1.

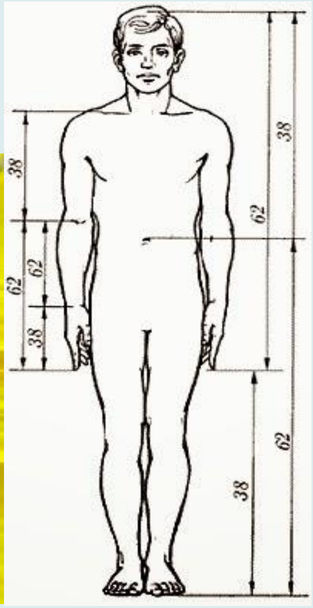
This project has been funded with support from the European Commission. This publication reflects the views only of the author, and the Commission cannot be held responsible for any use which may be made of the information contained therein.

Introduction

Though arts and math can seem to be unrelated on first sight, both can be said to be used by humans in their attempt to express physical and metaphysical reality. The Golden Ratio, also called Golden Proportion, is the oldest and strongest tie between mathematics and arts. Not only Ancient Greeks have been using it in architecture and arts, but it is also used very often nowadays. This is done in an attempt to reach harmony, and the golden ratio does exactly that, it is actually inextricably related to harmony. The Ancient Greeks, who were the first ones to develop the science of *aesthetics*, analyzed beauty, believing that harmony is its very basis. Beyond the meretricious use of the Golden Ratio in arts, the Golden Ratio can be found widespread in nature such as in plants, shells, flowers, animals and even in the proportion of human body.

Picture 1: Golden Ratio in Shells (Retrieved from: https://cdn.insteading.com/wp-content/uploads/igm/b/nautilus-shell.jpg)

Picture 2: Golden Ratio in Flowers (Retrieved from: https://www.reddit.com/r/sunflowers/duplicates/1vwwn3/natural_fractal_sunflower_spiral/)



Picture 3: Golden Ratio in Human body (Retrieved from: http://fitnessandhealthforyou.blogspot.com/2013/11/the-history-of-golden-ratio-and-adonis.html)

This project has been funded with support from the European Commission. This publication reflects the views only of the author, and the Commission cannot be held responsible for any use which may be made of the information contained therein.

> The term "proportion" is more relevantly used to compare relation between parts of things or to describe harmonious relation between different parts. (…)There are many well-known "formulas of beauty" such as certain geometrical shapes: square, circle, isosceles triangle, and pyramid. However, the most wide-spread criterion of beauty is one unique mathematical proportion called the Golden ratio which is termed as Divine proportion or Golden section or Golden number or Golden mean (Thapa and Thapa, 2018, p.190).

Legend has it that the Greek philosopher Pythagoras was the first one to discover the Golden ratio through the musical harmony/tuning produced, while listening to the different sounds of blacksmiths' hammers when beating their anvils. Subsequent to additional studies with stringed instruments and observing nature, Pythagoras concluded that the ratio of small integers defines beauty. Later, the Greek father of geometry Euclid was the first one to define Golden ratio as "A straight line is said to have been cut in extreme and mean ratio when, as the whole line is to the greater segment, so is the greater to the less" (cited by Thapa and Thapa, 2019).

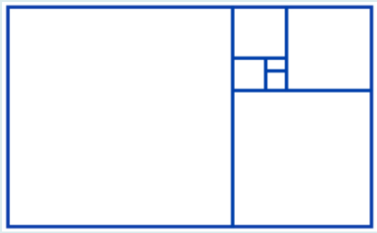
The Math Behind Arts and Architecture

Golden Ratio explained:

(i) Golden ratio explained:
https://www.youtube.com/watch?v=6nSfJEDZ_WM
https://www.youtube.com/watch?v=c8ccsE_IumM

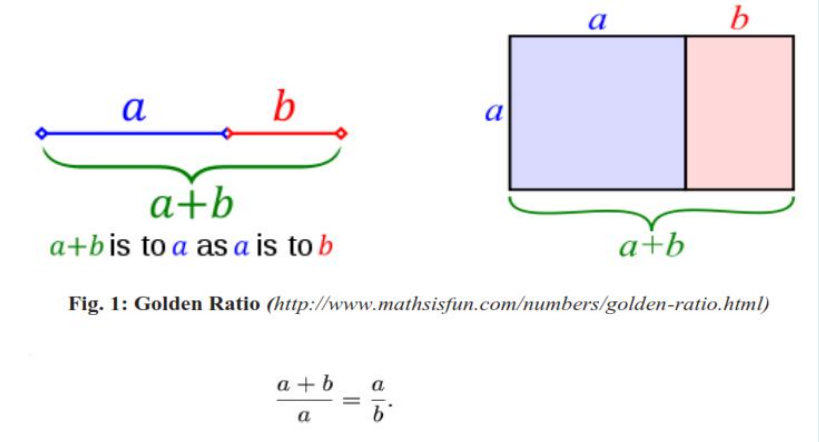
(ii) The golden ratio can be found depicted as a single large rectangle formed by a square and another rectangle. What is unique about this is that the sequence can be repeated infinitely and perfectly within each section.

This project has been funded with support from the European Commission. This publication reflects the views only of the author, and the Commission cannot be held responsible for any use which may be made of the information contained therein.



Picture 4: Sequence of square and rectangle (Retrieved from: http://files.voog.com/0000/0003/0740/files/Golden%20Section%20and%20Rabatment.pdf)

(iii) The idea behind the Golden ratio is: if a line is divided into two parts, the ratio of longer part and smaller part should be equal to the ratio of whole length and longer part. This makes the Golden ratio as in Picture 5.



Picture 5: Golden Ratio (Retrieved from: http://www.mathsisfun.com/numbers/golden-ratio.html)

The ratio $\frac{\mathbf{a}}{\mathbf{b}} = \frac{\mathbf{a+b}}{\mathbf{a}}$ is often represented with the Greek letter φ, so as:

$$\boldsymbol{\varphi} = \frac{\mathbf{a}}{\mathbf{b}} = \frac{\mathbf{a+b}}{\mathbf{a}}$$

Whereas φ is constant and equals to 1.618

Suggested Exercises:

1) Use the slider to try the Golden Ratio yourself : https://www.mathsisfun.com/numbers/golden-ratio.html

2) You can make Golden Rectangle by folding paper (a method called Origami) using these links https://www.youtube.com/watch?v=E6ioUH5tcbM

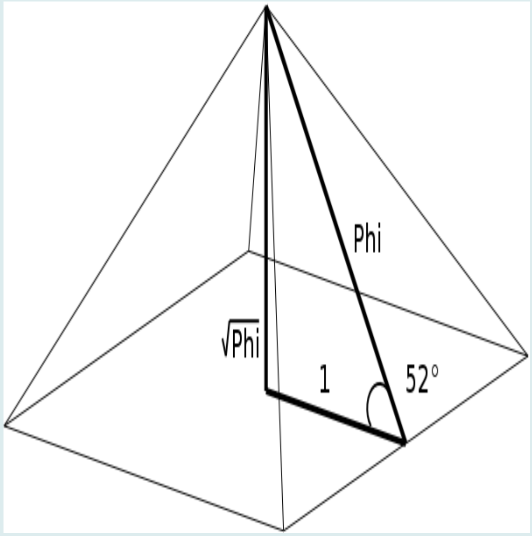
This project has been funded with support from the European Commission. This publication reflects the views only of the author, and the Commission cannot be held responsible for any use which may be made of the information contained therein.

The Golden Ratio behind Visual Arts

[Architectural designs and Paintings]

Ancient Architectural Designs

The first application of the Golden Ratio in architecture seems to go as far as 3000 BC, whilst several scholars seem to believe that the Egyptians applied the golden ratio to build the great pyramids of Giza. The length of each side of the base is 756 feet, and the height is 481 feet. So, we can find that the ratio of the vase to height is 756/481=1.5717.

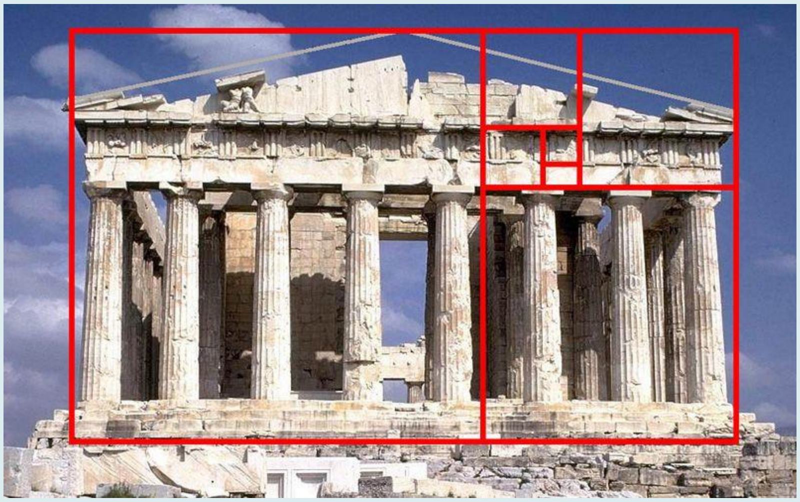


Picture 6: Golden Ratio in Pyramid shapes (Retrieved from: https://hbfs.wordpress.com/2009/12/08/cats-pharaohs-and-the-golden-ratio/)

Picture 7: Golden Ratio in Ancient Egyptian Pyramids (Retrieved from: https://twitter.com/intelsoftware/status/744201276729266176

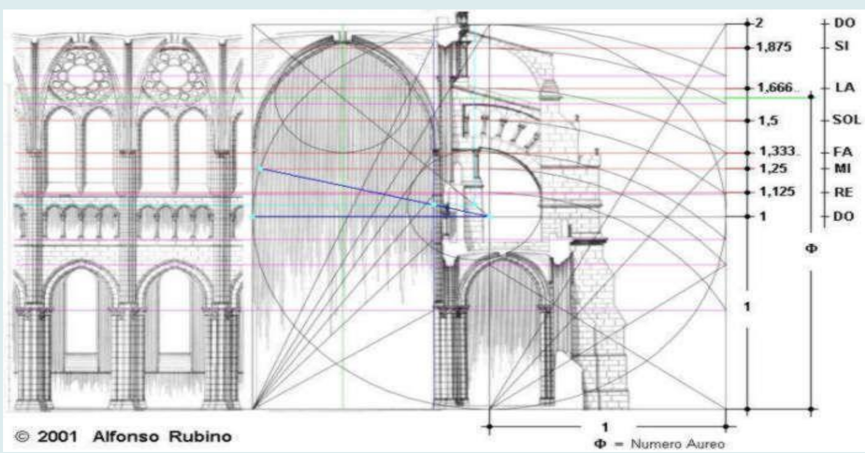
This project has been funded with support from the European Commission. This publication reflects the views only of the author, and the Commission cannot be held responsible for any use which may be made of the information contained therein.

The ancient Greeks also used the golden ratio when building the Parthenon.



Picture 8: The Golden Ratio in Parthenon (Retrieved from: https://www.pinterest.com/pin/302374562453891702/?lp=true)

Another example of the use of Golden ratio is in Medieval buildings of churches and cathedrals approaching the design of their buildings in a similar way as the Greeks. They tried to connect geometry and art. Both in and out, their buildings were complex constructions based on the golden section.



Picture 9: Chartre Cathedral (Retrieved from: https://www.pinterest.com/pin/309341068127486696/?lp=true)

This project has been funded with support from the European Commission. This publication reflects the views only of the author, and the Commission cannot be held responsible for any use which may be made of the information contained therein.

Contemporary Architectural Designs

Charles-Édouard Jeanneret (6 October 1887 – 27 August 1965), known as Le Corbusier, was a Swiss-French architect, designer, painter, urban planner, writer, and one of the pioneers of what is now called modern architecture. He was born in Switzerland and became a French citizen in 1930. His career spanned five decades, and he designed buildings in Europe, Japan, India, and North and South America. It is said that he used to arrange his designs using the Golden Ratio, mostly because of his absolute faith in science and mathematics, instead of a desire to achieve beauty.

Picture 10: Golden Ration in Le Corbusier's architecture (Retrieved from: http://jwilson.coe.uga.edu/emt668/emat6680.2000/obara/emat6690/Golden%20Ratio/golden.html)

Another famous architect from Switzerland, Mario Botta (the author of the old SF MoMA building) has also used this pattern to his designs.

This project has been funded with support from the European Commission. This publication reflects the views only of the author, and the Commission cannot be held responsible for any use which may be made of the information contained therein.

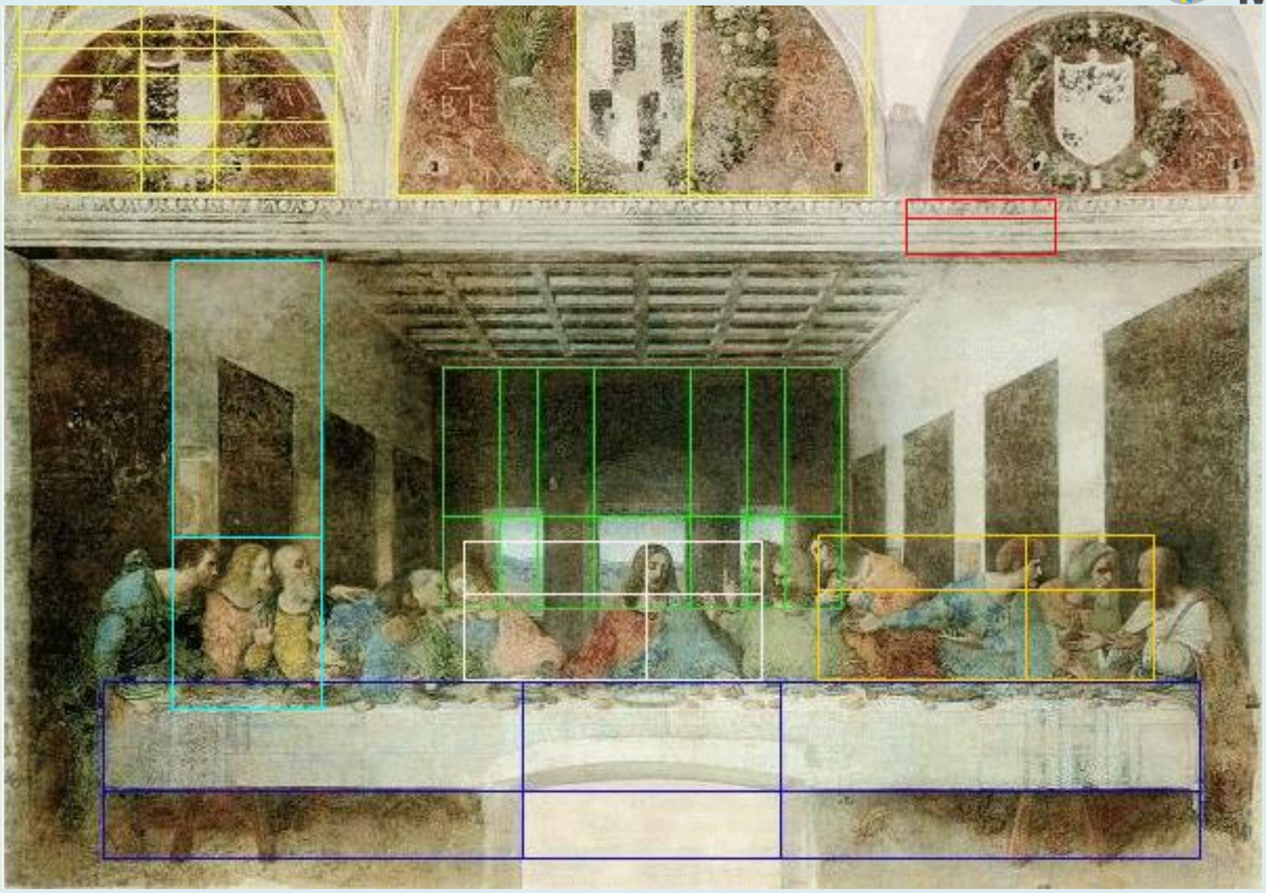
Picture 11: SF Moma (Retrieved from: https://www.widewalls.ch/golden-ratio-in-contemporary-architecture/)

Paintings using the Golden Ratio

Artists all over history, like Leonardo DaVinci, Botticelli and Salvador Dali, have used the golden ratio, the golden rectangle, or variations of it, as the center for their creations.

The Golden Section was used extensively by Leonardo Da Vinci. All the key dimensions of the room, the table and ornamental shields in Da Vinci's "The Last Supper" were based on the Golden Ratio.

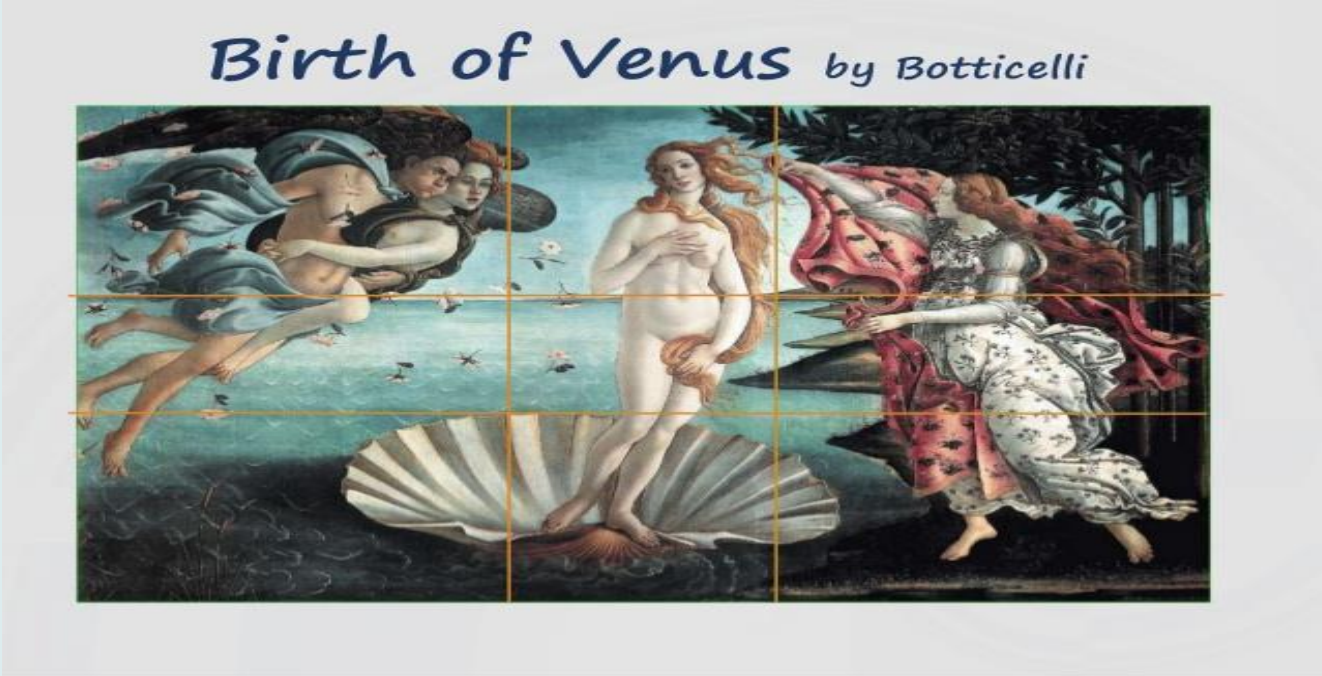
This project has been funded with support from the European Commission. This publication reflects the views only of the author, and the Commission cannot be held responsible for any use which may be made of the information contained therein.



Picture 12: Last Supper by Da Vinci (Retrieved from: https://www.goldennumber.net/art-composition-design/)

The Birth of Venus by Botticelli was completed in 1485. Botticelli made a number of paintings of the Annunciation between the years 1485 and 1490. This birth clearly captures the meeting of the divine with the mortal, and a brilliant opportunity to apply the Divine Proportion.

This project has been funded with support from the European Commission. This publication reflects the views only of the author, and the Commission cannot be held responsible for any use which may be made of the information contained therein.



Picture 13: Birth of Venus by Botticelli (Retrieved from: https://www.slideshare.net/TatyanaSerova/golden-ratio-55052186)

The sacrament of the Last Supper by Salvador Dali(1904-1989). This picture is painted inside a Golden Rectangle. Also, we can find part of an enormous dodecahedron above the table. Since the polyhedron consists of 12 regular Pentagons, it is closely connected to the golden section.

Picture 14: Last Supper by Salvador Dali **(**Retrieved from: https://slideplayer.com/slide/6024055/)

This project has been funded with support from the European Commission. This publication reflects the views only of the author, and the Commission cannot be held responsible for any use which may be made of the information contained therein.

Glossary

Pythagoras: Pythagoras of Samos[a] (c. 570 – c. 495 BC) was an ancient Ionian Greek philosopher and the eponymous founder of Pythagoreanism. His political and religious teachings were well known in Magna Graecia and influenced the philosophies of Plato, Aristotle, and, through them, Western philosophy. In antiquity, Pythagoras was credited with many mathematical and scientific discoveries, including the Pythagorean theorem, Pythagorean tuning, the five regular solids, the Theory of Proportions, the sphericity of the Earth, and the identity of the morning and evening stars as the planet Venus. It was said that he was the first man to call himself a philosopher ("lover of wisdom")and that he was the first to divide the globe into five climatic zones.

Retrieved from: https://en.wikipedia.org/wiki/Pythagoras

Euclid: (300 BC), was a Greek mathematician, often referred to as the "founder of geometry" or the "father of geometry". He was active in Alexandria during the reign of Ptolemy I (323–283 BC). His Elements is one of the most influential works in the history of mathematics, serving as the main textbook for teaching mathematics (especially geometry) from the time of its publication until the late 19th or early 20th century. The English name Euclid is the anglicized version of the Greek name Εὐκλείδης, which means "renowned, glorious".

Retrieved from: https://en.wikipedia.org/wiki/Pythagoras

This project has been funded with support from the European Commission. This publication reflects the views only of the author, and the Commission cannot be held responsible for any use which may be made of the information contained therein.

Erasmus+

TheArtOf Maths

TASKS

TASK 1

(a) Considering that $\boldsymbol{\varphi} = \frac{\mathbf{a}}{\mathbf{b}} = \frac{\mathbf{a+b}}{\mathbf{a}}$, prove that this is equivalent to $\boldsymbol{\varphi} = \mathbf{1} + \mathbf{1}/\boldsymbol{\varphi}$

(b) Use $\boldsymbol{\varphi} = \mathbf{1} + \mathbf{1}/\boldsymbol{\varphi}$ to calculate $\boldsymbol{\varphi}$

TASK 2

Follow the guidelines in order to draw a rectangle with Golden Ratio:

a) Draw a square

b) Consider the length of the side as 1

c) With your pencil, mark the middle of the one side (preferably the one which is at the bottom), to divide it to two equal parts of size ½

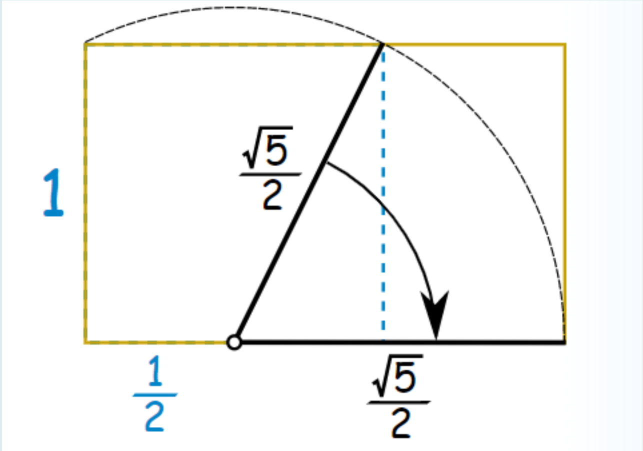
d) Now, draw a line from the middle point you have marked to one of the opposite corners (either left or right).

e) Use your previous mathematical knowledge to estimate the length of this line.

f) Subsequently, by using your compass, turn the line in a way that it turns along the square's side

g) Extent the square, to create a 'golden ratio' rectangle. The result is depicted within the following picture,

This project has been funded with support from the European Commission. This publication reflects the views only of the author, and the Commission cannot be held responsible for any use which may be made of the information contained therein.



h) Could you now calculate φ from the picture?

This project has been funded with support from the European Commission. This publication reflects the views only of the author, and the Commission cannot be held responsible for any use which may be made of the information contained therein.

LEARN MORE...

If you want to further investigate on the topics addressed in this tool, you may go through the following links:

What is Golden Mean and Golden Ratio:

https://www.youtube.com/watch?v=6nSfJEDZ_WM

https://www.mathsisfun.com/numbers/golden-ratio.html

TED TALK: 'The ab(surb) Golden Ratio'

https://www.youtube.com/watch?v=0vVxL60YFJU

ArtNews:

https://news.artnet.com/art-world/golden-ratio-in-art-328435

A Guide to the Golden Ratio (AKA Golden Section or Golden Mean) for Artists:

https://emptyeasel.com/2009/01/20/a-guide-to-the-golden-ratio-aka-golden-section-or-golden-mean-for-artists/

The golden ratio in pyramids:

https://www.goldennumber.net/great-pyramid-giza-complex-golden-ratio/

The Importance of Golden Ratio in Contemporary Architecture

https://www.widewalls.ch/golden-ratio-in-contemporary-architecture/

This project has been funded with support from the European Commission. This publication reflects the views only of the author, and the Commission cannot be held responsible for any use which may be made of the information contained therein.

Botticelli and the use of Golden Ratio:

https://www.goldennumber.net/botticelli-birth-venus-golden-ratio-art/

Journal articles:

Thapa, G. B., & Thapa, R. (2018). The relation of golden ratio, mathematics and aesthetics. Journal of the Institute of Engineering, 14(1), 188. Retrieved from http://search.ebscohost.com/login.aspx?direct=true&AuthType=ip,sso&db=edb&AN=132701041&site=eds-live&custid=s1098328

Obara, Samuel. Golden Ratio in Art and Architecture. University of Georgia, 2000. Web. http://jwilson.coe.uga.edu/EMT668/EMAT6680.2000/Obara/Emat6690/Golden%20Ratio/golden.html . Retrieved: 18th June 2019

This project has been funded with support from the European Commission. This publication reflects the views only of the author, and the Commission cannot be held responsible for any use which may be made of the information contained therein.