PART IV: Cinematography & Mathematics

AGE RANGE: 16-18

TOOL 40: APPROACHING PRIME NUMBER THEORY THROUGH THE MOVIE "THE MAN WHO KNEW INFINITY"

C.I.P. Citizens In Power

The Art Of Maths

Co-funded by the Erasmus+ Programme of the European Union

Educator's Guide

Title: Approaching The Prime Number theory and Partitions through the movie 'The man who knew infinity'

Age Range: 16-18 years old

Duration: 1.5 hour

Mathematical Concepts: Prime Number Theory and Partitions

Artistic Concepts: Cinematography

General Objectives: Mathematical wise students will familiarize with the prime numbers and partitions, use the mathematical examples given, to analyze the task and learn (or at least familiarize with) the formula of partitions. They will also see some of the steps that a mathematical research involves. In the art behind, they will have the chance to see the vast number of mathematical movies there are and get acquaintance with Ramanujan, his work and his biography, through pictures, videos and some literacy fragments.

Instructions and Methodologies: The methodologies used here follow Blooms' Taxonomy, starting with the knowledge of who Ramanujan was; a reminder of what the prime numbers and the partitions are, to a more comprehensive level of explaining them. Then they will apply their own small formula through the task given for the ultimate goal of being able to use their knowledge on partitions.

Resources: Youtube videos consisting of a synopsis of the actual life of Srinivasa Ramanujan and excerpts of the film *'The man who knew infinity'*. There are some pictures; Ramanujan's biography and examples of partitions; the task itself, Ramanujan's and a youtube video explaining it.

Tips for the educator: This tool starts by giving some information through Youtube videos, pictures and the biography of the mathematician, before associating it with the film and elements from it. It will be important to grasp the interest of your students by emphasizing the difficulties Ramanujan was phasing in his time and real life (including poverty and obstacles due to his Indian origin). It is a good idea to also emphasize on elements of someone's character as an example/role model to students, which helped Ramanujan excel, beyond his prodigy mind, like his

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persistency, hard work and devotion which ultimately helped him remain in history (these elements are obvious through the video link on his biography).

Desirable Outcomes and Competences: Students will:

- identify who this great mathematician was (biographical elements);
- experiment with their own formula of partitions.

Debriefing and Evaluation Questions:

You can use these cards sometimes called EXIT CARDS either by a hard copy made from before or simply by posing these statements on board and the students write their answers on a paper which they will leave preferably anonymously while exiting the room. The specific formative strategy is called 3,2,1. For more strategies you can visit:

https://www.bhamcityschools.org/cms/lib/AL01001646/Centricity/Domain/131/70%20Formative%20Assessments.pdf

3-2-1	
Write 3 things you liked about this activity	**1.** **2.** **3.**
Write 2 things you have learned	**1.** **2.**
Write 1 aspect for improvement	**1.**

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Introduction

According to Polster (2012) there are more than 700 mathematical movies although some related to mathematics to a very big extent and some to a much smaller; they are considered as an injection of moments of fun, which can be used into courses in an attempt to make the learning of mathematics more interesting for the young audiences. For this task, the movie '*The man who knew infinity*' which is based on the homonymous Book by Robert Kanigel, has been chosen for several reasons.

Firstly, it is one of the movies alertly related to mathematics and the story of a great Indian mathematician of the 20th century, called Srinivasa Ramanujan. Also, the film has to offer great insights presenting mathematics as art but also as a creative process of discovery, lending into several mathematical concepts and mostly, prime numbers and partitions. The movie also gives great implications on ones character as a role model of young adults.

The film captivates the grasps of what it means to undertake mathematical research. The protagonist is mostly urged by curiosity and tries to capture the striking and graceful connections among abstract concepts. These explorations naturally involve some sort of experimentation, but relying mostly on ideas and symbols instead of physical things. As we can see both in the book and the movie, there are lots of mistakes and dead ends. Thus, more persistency is needed. That is why when the character enters the typical education of the English University, he is required to give proofs –complete, verifiable, logical justifications– of his assertions. Constructing the proof can be difficult and often takes a lot longer than the initial discovery.

What is emphasized through the film, and an obligation of real mathematical research, is to avoid the temptation even of such great minds to move from discovery to discovery, from one connection to another, before giving the proofs to support the ones already found. Tertiary education in mathematics aims to instill this. In India, Ramanujan lacked such an education. At Cambridge, he had to catch up and fill those gaps.

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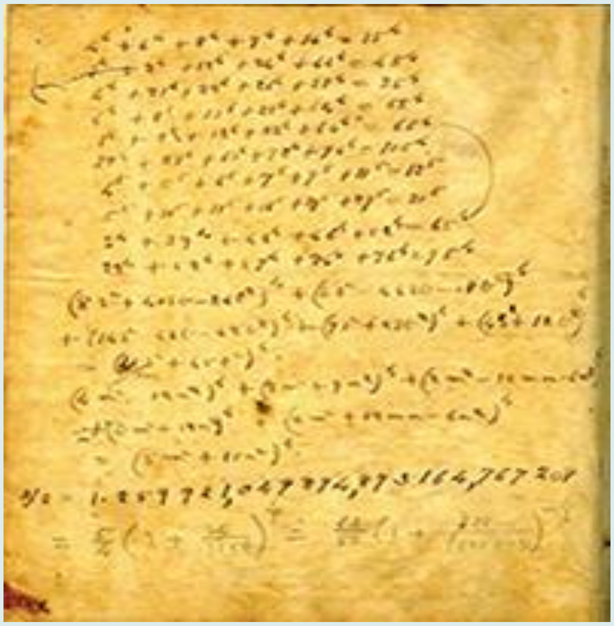
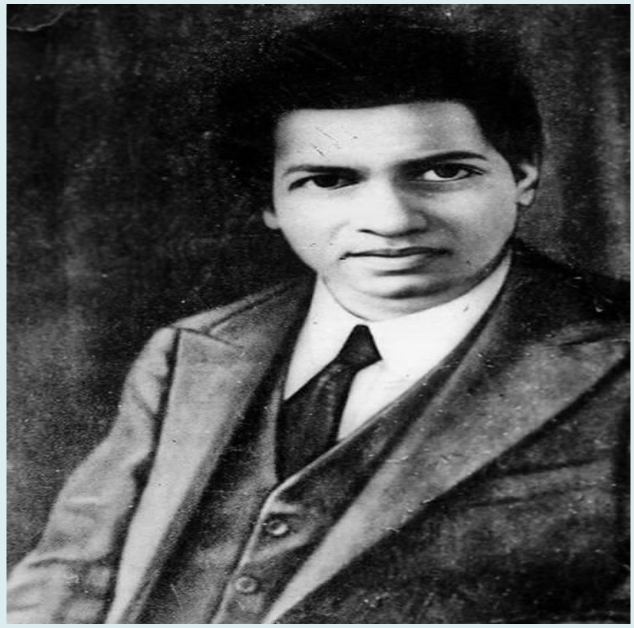
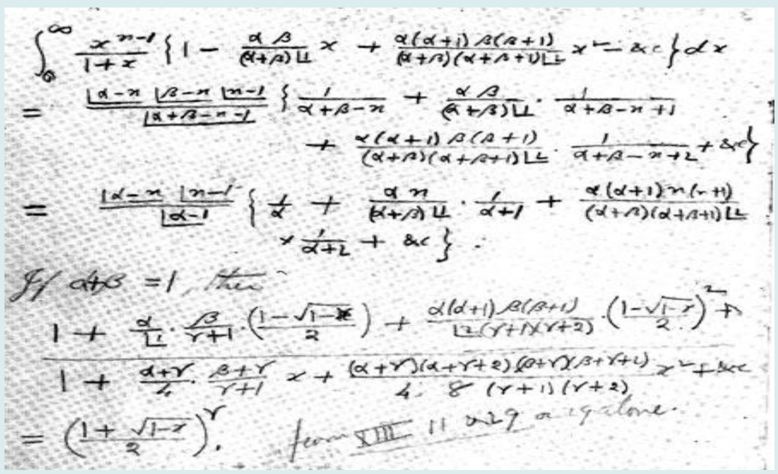
Biography



Picture 1: original notes of Rmanujan [1]



Picture 2: Rmanujan [2]



Picture 3: Ramanujan's notes[3]

[1] Retrieved from: https://www.google.com/search?q=notebooks+of+ramanujan+pdf&client=firefox-b-d&source=lnms&tbm=isch&sa=X&ved=0ahUKEwiZn5uFtd7iAhWNyKQKHVdoD10Q_AUIECgB&biw=1138&bih=527#imgdii=CdWIT6ACYDdArM:&imgrc=dNSzdmvpv-YSrM:

[2] Retrieved from: https://www.google.com/search?client=firefox-b-d&biw=1138&bih=527&tbm=isch&sa=1&ei=awr-XNjHFcGMa72wosgC&q=ramanujan&oq=raman&gs_l=img.1.0.0i67l2j0j0i67j0l2j0i67j0l2j0i67.168570.169133..172117...0.0..0.189.834.0j5......0....1..gws-wiz-img.......35i39.CT6QYjVtbPE#imgrc=9Rr_y3zQwk-1TM:

[3] Retrieved from: https://www.google.com/search?client=firefox-b-d&biw=1138&bih=527&tbm=isch&sa=1&ei=ABP-XKLMCszLwQL6-ZToCw&q=ramanujan%27s+notebooks&oq=ramanujan%27s+notebooks&gs_l=img.3.0.0j0i5i30j0i8i30j0i24l2.

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Srinivasa Ramanujan (22 December 1887 – 26 April 1920) was an Indian mathematician who lived during the British Rule in India. Though he had almost no formal training in pure mathematics, he made substantial contributions to mathematical analysis, number theory, infinite series, and continued fractions, including solutions to mathematical problems then considered to be unsolvable. Ramanujan initially developed his own mathematical research in isolation. Seeking mathematicians who could better understand his work, in 1913 he began a postal partnership with the English mathematician G. H. Hardy at the University of Cambridge, England. In his notes, Ramanujan had produced groundbreaking new theorems, including some that Hardy stated had "defeated [him and his colleagues] completely", in addition to rediscovering recently proven but highly advanced results.

During his short life, Ramanujan independently compiled nearly 3,900 results (mostly identities and equations). Many were completely novel; his original and highly unconventional results, such as the Ramanujan prime, the Ramanujan theta function, partition formulae and mock theta functions, have opened entire new areas of work and inspired a vast amount of further research. Nearly all his claims have now been proven correct. The Ramanujan Journal, a peer-reviewed scientific journal, was established to publish work in all areas of mathematics influenced by Ramanujan, and his notebooks—containing summaries of his published and unpublished results—have been analyzed and studied for decades since his death as a source of new mathematical ideas.

In 1919, ill health, now believed to have been hepatic amoebiasis, compelled Ramanujan's return to India, where he died in 1920 at the age of 32.

Information retrieved from Wikipedia: https://en.wikipedia.org/wiki/Srinivasa_Ramanujan

39595.44881..50313...0.0..0.154.2622.0j21......0....1..gws-wiz-img.......35i39j0i67j0i30.GMm5Q9WIy7M#imgrc=omvjbPsONG-PZM

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Plot of the movie 'The man who knew infinity'

At the 20th century, Srinivasa Ramanujan is an under pressure and poor citizen living in India, and specifically in the city of Madras, working at unskilled jobs at the border of poverty. While working, his employers become aware of his exceptional skills in mathematics and they start using him for basic tasks in accounting. After his employers realize that his mathematical insights exceed the simple accounting tasks, they shortly encourage him to make his own writings in mathematics available to the public and to begin to contact mathematics professors at universities outside India. One of those letters is sent to G.H. Hardy, a famous mathematician at University of Cambridge, who starts taking particular interest in Ramanujan.

Ramanujan at the same time gets married while working and sending out his first publications. Hardy almost immediately invites Ramanujan to Cambridge to assess his determination as a possible theoretical mathematician. Ramanujan is excited by the opportunity and decides to chase Hardy's offer, although this means he must leave his wife for a long period. He parts tenderly with his wife and promises to keep writing letters to her.

As soon as he arrives to Cambridge, Ramanujan faces various forms of racial chauvinism and finds the alteration to life in England more difficult than expected. Although Hardy is much impressed by Ramanujan's abilities, he is worried about Ramanujan's capability to speak effectively due to his lack of experience in writing proofs, but with determination he manages to get Ramanujan published in a major journal. In the meantime, Ramanujan finds out that he is suffering from tuberculosis whilst his regular letters to his wife remain unanswered after many months. Hardy, although keep seeing much promise in Ramanujan, he remains unaware of the personal difficulties he is facing considering his health and the lost of communication with his wife. Ramanujan's health worsens even as he continues delving into deeper and more profound research interests in mathematics under the supervision of Hardy and others colleagues at Cambridge.

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His wife eventually discovers that his mother has been hiding his letters and not sending hers to him. Hardy tries to get Ramanujan's recognizably exceptional mathematical skills to be totally accepted by his university through nominating Ramanujan for a fellowship at Trinity College. At first, Hardy fails due to politics related to the college and persistent racial prejudice of the time. Later, though by gaining the support of key members of the college, Hardy once more nominates Ramanujan for a fellowship; and he is finally accepted as a Fellow of the Royal Society and then as a Fellow of Trinity College. Ramanujan is at the end reunited with his family in India, though his waning health, which suffered mostly from poor housing and harsh winter weather in England, ultimately leads to his premature death as soon after his recognition as a mathematician of international merit and importance.

Excerpt from the movie

Official trailer: https://www.youtube.com/watch?time_continue=146&v=oXGm9VIfx4w

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The Math behind the movie 'The man who knew Infinity'

Ramanujan's Formula

The first simple formula has been found for calculating how many ways a number can be created by adding together other numbers, solving a puzzle that captivated the legendary mathematician Srinivasa Ramanujan.

The feat has also led to a greater understanding of a cryptic phrase Ramanujan used to describe sequences of so-called partition numbers.

A partition of a number is any combination of integers that adds up to that number. For example, 4 = 3+1 = 2+2 = 2+1+1 = 1+1+1+1, so the partition number of 4 is 5. It sounds simple, yet the partition number of 10 is 42, while 100 has more than 190 million partitions. So a formula for calculating partition numbers was needed.

Previous attempts have only provided approximations or relied on "crazy infinite sums", says Ken Ono at Emory University in Atlanta, Georgia.

Pattern in partition

Ramanujan's approximate formula, developed in 1918, helped him spot that numbers ending in 4 or 9 have a partition number divisible by 5, and he found similar rules for partition numbers divisible by 7 and 11.

Without offering a proof, he wrote that these numbers had "simple properties" possessed by no others. Later, similar rules were found for the divisibility of other partition numbers so no one knew whether Ramanujan's words had a deeper significance.

Now Ono and colleagues have developed a formula that spits out the partition number of any integer. They may also have discovered what Ramanujan meant.

They found "fractal" relationships in sequences of partition numbers of integers that were generated using a formula containing a prime number. For example, in a

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sequence generated from 13, all the partition numbers are divisible by 13, but zoom in and you will find a sub-sequence of numbers that are divisible by 132, a further sequence divisible by 133 and so on.

By Jacob Aron, retrieved from: https://www.newscientist.com/article/dn20039-deep-meaning-in-ramanujans-simple-pattern/

Video explaining the formula:

https://www.youtube.com/watch?v=nxdGOLp56nc

Glossary

Prime number: A prime number is a whole number greater than 1 whose only factors are 1 and itself. A factor is a whole numbers that can be divided evenly into another number. The first few prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23 and 29. Numbers that have more than two factors are called composite numbers. The number 1 is neither prime nor composite.

Partition: In number theory and combinatorics, a partition of a positive integer n, also called an integer partition, is a way of writing n as a sum of positive integers. Two sums that differ only in the order of their summands are considered the same partition. (If order matters, the sum becomes a composition.) A summand in a partition is also called a part. The number of partitions of n is given by the partition function p(n). So p(4) = 5. The notation $\lambda \vdash n$ means that λ is a partition of n. Partitions can be graphically visualized with Young diagrams or Ferrers diagrams. They occur in a number of branches of mathematics and physics, including the study of symmetric polynomials and of the symmetric group and in group representation theory in general.

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Example

The seven partitions of 5 are:

- 5
- 4 + 1
- 3 + 2
- 3 + 1 + 1
- 2 + 2 + 1
- 2 + 1 + 1 + 1
- 1 + 1 + 1 + 1 + 1

In some sources partitions are treated as the sequence of summands, rather than as an expression with plus signs. For example, the partition 2 + 2 + 1 might instead be written as the tuple (2, 2, 1) or in the even more compact form (22, 1) where the superscript indicates the number of repetitions of a term.

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TASK

The partition function p(n) represents the number of possible partitions of a non-negative integer. For instance p(4)=5, because the integer 4 has the five partitions:

- 1+1+1+1;
- 1+1+2;
- 1+3;
- 2+2;
- and 4

Given this, estimate the sum S= p(4) + p(6) + p(8)

Video explaining Ramanujan's formula:

https://www.youtube.com/watch?v=nxdGOLp56nc

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LEARN MORE…

If you want to further investigate on the topics addressed in this tool, you may go through the following links:

Book on math related to movies:
Polster, B., & Ross, M. (2012). *Math goes to the movies*. Baltimore: Johns Hopkins University Press. Retrieved from http://search.ebscohost.com/login.aspx?direct=true&AuthType=ip,sso&db=nlebk&AN=597694&site=eds-live&custid=s1098328

Ramanujan's Biography:
https://en.wikipedia.org/wiki/Srinivasa_Ramanujan

What is a prime number:
https://whatis.techtarget.com/definition/prime-number

What are partitions:
https://en.wikipedia.org/wiki/Partition_(number_theory)

Ramanujan's Formula:
https://www.newscientist.com/article/dn20039-deep-meaning-in-ramanujans-simple-pattern/

The Man Who Knew Infinity: inspiration, rigor and the art of mathematics
May 24, 2016:
https://theconversation.com/the-man-who-knew-infinity-inspiration-rigour-and-the-art-of-mathematics-59520

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