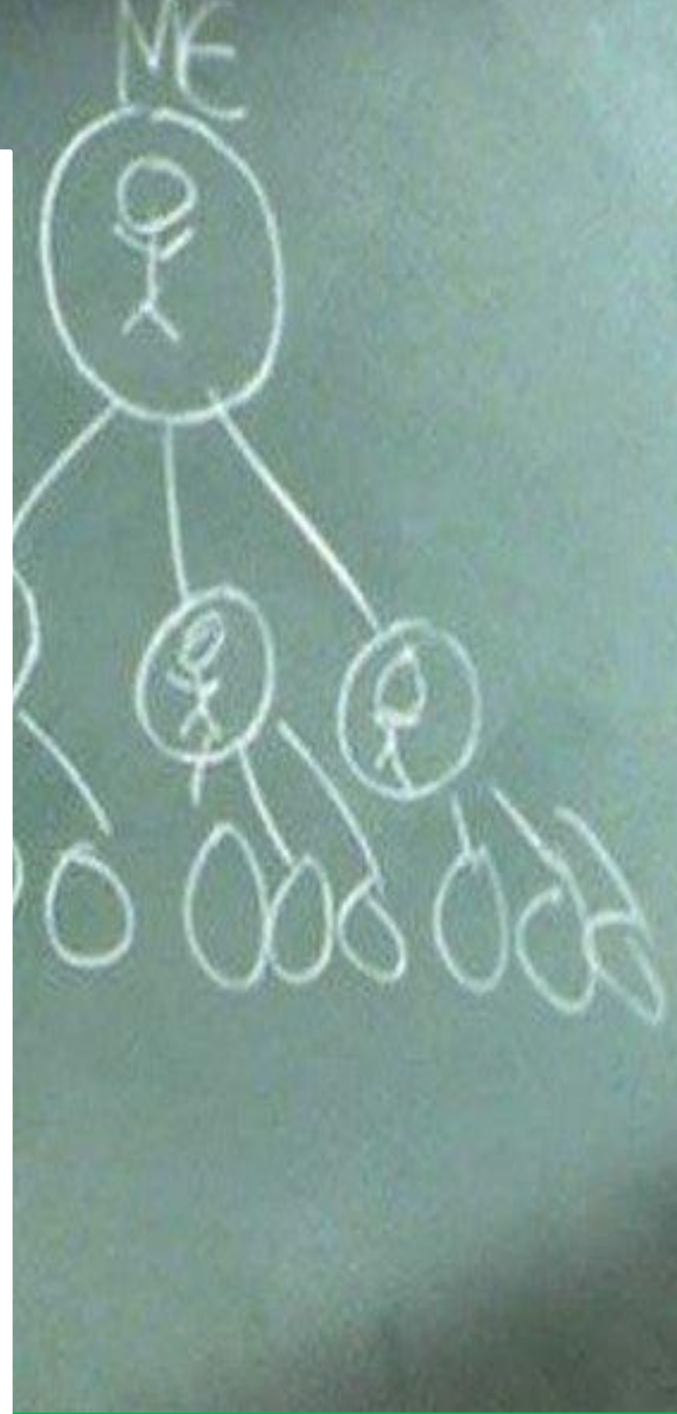
PART IV: Cinematography & Mathematics

AGE RANGE: 16 – 18



Pay it Forward infographic.
(Source: https://cinemaplanet.pt/pay-it-forward-boa-acao-vez/)

TOOL 39: EXPONENTIAL GROWTH THROUGH THE MOVIE "PAY IT FORWARD"

SPEL – Sociedade Promotora de Estabelecimentos de Ensino

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Educator's Guide

Title: Exponential Growth through the movie "Pay it Forward"

Age range: 16 –18 years old

Duration: 2 hours

Mathematical concepts: Exponent, Exponential functions, Exponential Growth, Exponential Decay, Exponents

Artistic concepts: Exponential model

General objectives: To study and analyse real-life examples of exponential growth and decay; To graph exponential equations and functions.

Instructions and Methodologies: Show the trailer of the movie Pay it Forward (cf. link on "Searching the NET") and suggest students to watch the full movie at home;

Resources: A pen and a calculator.

Tips for the educator: To help students understand the results from the graphs, let them propose different values and make their graphical representations.

Learning Outcomes and Competences: At the end of this tool, the student will be able to:

- Recognize and solve problems involving the application of exponential functions, as well as a graphical representation that is likely to represent an exponential functions;
- Use the rules of exponents to work with exponential functions and make their graphical representation.

Debriefing and Evaluation:

Write 3 aspects you liked about this activity:	1. 2. 3.
Write 2 aspects that you have learned	1. 2.
Write 1 aspect for improvement	1.

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Introduction

Sometimes we find aspects related to Mathematics in television series or movies. In such cases, sometimes these Mathematical concepts are not given much importance, because they do not influence the story itself. However, there are a few cases in which they do.

Some examples include: "21" (USA, 2008), by Robert Luketic; "Proof" (USA, 2005), by John Madden; "A Beautiful Mind" (USA, 2001), by Ron Howard; "Enigma" (USA, 2001), by Michael Apted; "Pi" (USA, 1988), by Darren Aronofsky; "Good Will Hunting" (USA, 1997), by Gus Van Sant and "Cube" (Canada, 1997), by Vincenzo Natali.

In this tool, the movie “Pay it forward” (USA, 2000), by Mimi Leder, will be discussed and its mathematical concepts, such as exponential growth, will be covered.

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Pay it Forward

Following a Social Studies class assignment to change the world for the better, 12-year-old student Trevor McKinnney (played by Haley Joel Osment) starts a movement that consists in making a good deed for three people whom, instead of paying the favour back are told to "pay it forward" by doing a favour to three other people, and so forth.

On his quest to change the world, Trevor eventually triggers a series of events that can be represented as an exponential model, meaning that these movements meet the needed requirements to grow at an exponential rate overtime.

Fig. 1 – Pay it Forward (2000) movie poster
(Source: https://en.wikipedia.org/wiki/Pay_It_Forward_(film))

The exponential model is associated with British economist Thomas Robert Malthus (1766-1834), who first noticed that any species could potentially grow according to a geometric series. Exponential functions derived from this observance.

In mathematics, exponential functions are functions that grow by common factors over equal intervals. This growth can result in an increase or decrease of the value, which, therefore, is known as exponential growth and exponential decay, respectively.

In the long term, the growth or decay becomes unsustainable, since the nutrients or resources will exhaust and the growth/decay will stop. Nonetheless, the use of the exponential functions to predict an outcome is often used in applications in business, science and sociology and can be witnessed in nature itself.

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Glossary

Exponent: corresponds to the repeated multiplication of a base "n": that is, the product of bn is the multiplication of the base (b) "n" times; called "b raised to the power of n" or "b raised to the nth power".

Exponential function: a function of the form $\boldsymbol{f(x) = a * b^x}$, in which the result changes exponentially as x increases.

Exponential growth: a concept used when a value increases proportionally at a constant rate over a period.

Exponential decay: a concept used when a value decreases proportionally at a constant rate over a period.

Factor: a number that, when divided by another number, results in an evenly number – i.e., with no decimals.

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The Math behind Pay it Forward

1. Exponential Functions

Exponential functions are characterized for having **a value that grows based on a common factor over an equal period**, whether it is an increase or a decrease of a certain value.

An exponential function is represented as:

Exponential function

$$\mathbf{y = a^x}$$

Where:

a = the **initial value** before measuring growth/decay and is **> 0**

x = a variable exponent that corresponds to the number of time intervals that have passed.

In the movie "Pay it Forward", Trevor starts a movement that could trigger actions that would triple exponentially.

By looking at the table of values below, let us analyze the potential of this impact by considering that, for every day passed, every person who has been involved in the movement will continue its process by making a good deed to 3 other people:

Day	Number of people	Pattern
1	3 = 3	$y = 3^1$
2	9 = 3x(3)	$y = 3^2$
3	27 = 3x(3x3)	$y = 3^3$
4	81 = 3x(3x3x3)	$y = 3^4$
5	243 = 3x(3x3x3x3)	$y = 3^5$

On the 1st day, Trevor does 3 good deeds to 3 different people; On the 2nd day, each of the 3 people do another 3 good deeds to 3 other people, making a total of 9 (3x3). By the 5th day, 243 people (3x3x3x3x3) will have been involved. In other words, the amount triples every day!

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This progress of each change can then be depicted through a graphic, as follows:

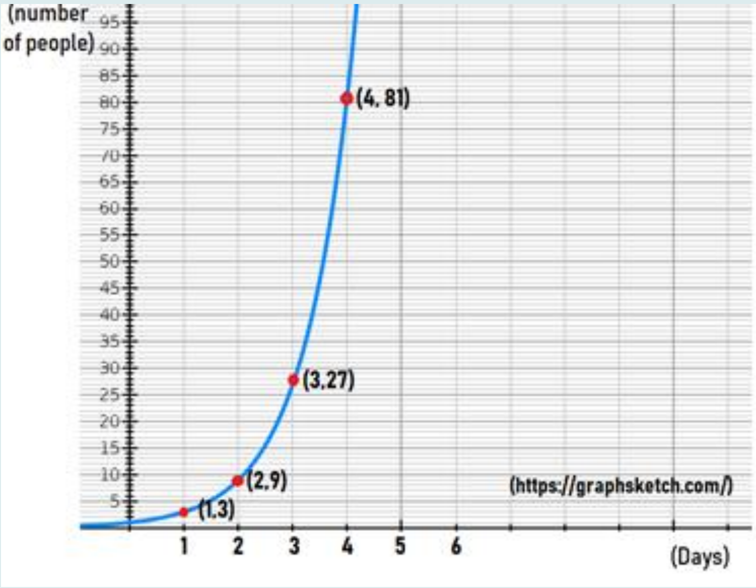

Fig. 2 – Estimation of people involved in Trevor's movement
(Source: Made by Author on graphsketch.com)

Exponential Growth and Decay

We have seen that exponential functions grow by common factors. However, if we want to introduce decimals instead, then Exponential Growth and Exponential Decay functions have to be invoked:

Exponential Growth	Exponential Decay
$\mathbf{y = a\,(1 + r)\,x}$	$\mathbf{y = a\,(1 - r)\,x}$

Where: **a** = the **initial value** before measuring growth/decay;

r = the constant **rate** at which the initial value will change; usually represented as a percentage and expressed as a decimal;

x = a variable exponent that corresponds to the number of time intervals that have passed.

When: **a > 0** and **r > 0**, there is an exponential growth;

a > 0 and **0 < r < 1**, there is an exponential decay.

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Exponential growth is often used to predict the growth of a population or interest compound. For instance, say there is a population of 2579 people in a nearby small town, and that every year there is a 12% growth. **How many people would there be after 5 years?**

Let us first model the function according to the question. We know that:

1) 2579 is our starting point;

2) There is a 12% or 0,12 growth per year;

So the function that will be used is $\mathbf{P(5) = 2579(1 + 0{,}12)^5}$, where "P(5)" corresponds to the population in 5 years.

As for de calculation:

$\mathbf{P(5) = 2579(1 + 0{,}12)^5 \Leftrightarrow P(5) = 2579(1{,}12)^5 \Leftrightarrow P(5) = 4545}$

Answer: In 5 years, there will be a population of approximately 4545 people.

Another real-life context in which exponential functions are applied is in Physics. Radioactive substances are atoms with unstable nucleus that decay naturally. In this case, **exponential decay** is used to predict how long a substance will be left or last after a period.

Consider a sample of 10mg of substance YXZ that decays 0,3% yearly. **How much of the sample would be left after 8 years?**

Let us first model the function according to the question. We know that:

1) 10mg is our starting point;

2) There is a 0,3% or 0,03 decay per year;

So, the function that will be used is $\mathbf{P(8) = 10(1 - 0{,}03)^8}$, where "P(8)" corresponds to the amount that will be left after 8 years have gone by.

As for de calculation:

$\mathbf{P(8) = 10(1 - 0{,}03)^8 \Leftrightarrow P(5) = 10(0{,}97)^8 \Leftrightarrow P(5) = \sim 7{,}83}$

Answer: In 8 years, there will be approximately 7,83mg of substance YXZ left.

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TASKS

TASK 1

Bacteria are single cell microbes that reproduce by splitting into two cells.

Consider a population of 20 cells that double every 1 minutes.

1.1) Create a table of values and calculate how many cells would there be in a period of 4 hours.

1.2) Depict the progress on a graphic.

TASK 2

Imagine that you invest €2000 at a monthly interest rate of 1,5%.

2.1) What would your balance be after half a year? Find the exponential growth function and calculate it.

TASK 3

In the previous estimation of people involved in the movement that Trevor started, we have seen that by day 4, if uninterrupted, there would have been 81 people involved.

Consider that, for some reason, at the end of day 4, the movement was to be suspended for 5 days and that, for each of those 5 days, there would be a decrease of 10% of people to pay the favour forward when the movement resumed.

3.1) How many people would be left to resume the movement by the end of day 9? Find the exponential decay function and calculate it.

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LEARN MORE…

Pay it Forward (2000) movie plot

https://www.imdb.com/title/tt0223897/?ref_=nv_sr_1?ref_=nv_sr_1

Pay it Forward Trailer

https://www.youtube.com/watch?v=qfW0wCV9iFI

Introduction to Exponential Functions

https://www.khanacademy.org/math/algebra/introduction-to-exponential-functions/exponential-vs-linear-growth/v/exponential-growth-functions

Exponential Equations: Exponential Growth and Decay Application

http://www.softschools.com/math/algebra/topics/exponential_equations_exponential_growth_and_decay_application/

Features of Exponential Functions

https://mathbitsnotebook.com/Algebra1/FunctionGraphs/FNGTypeExponential.html

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