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PART IV: Cinematography \& Mathematics

AGE RANGE: 13-15

TOOL 37: Coordinate System through the movie "Kingdom of Heaven"

SPEL - Sociedade Promotora de Estabelecimentos de Ensino


## Educator's Guide

Title: Coordinate system through the movie "Kingdom of Heaven"
Age range: 13-15 years old
Duration: 2 hours
Mathematical concepts: Cartesian coordinate plane
Artistic concepts: Battleship in the Cartesian plane
General objectives: To understand how to use a Cartesian coordinate plane, how to score points in it and how to read the coordinates of points presented.

Instructions and Methodologies: Besides the explanation of the theoretical concepts, it is important to use videos and texts for better understanding the contents.

Resources: Computer with an internet connection; access to the website: https://www.desmos.com/; board of the Battleship game.
Tips for the educator: Start by explaining the theoretical concepts and, if possible, use images, texts and videos for a more effective explanation. Proceed with giving various examples of point scoring and coordinate reading, to explain how to do it, so that the students can then solve the exercises on their own. Guide the students in the Battleship game, helping them understand its rules.
Learning Outcomes and Competences: At the end of this tool, the student will be able to:

- Use a Cartesian coordinate plane;
- Score points in a Cartesian coordinate plane;
- Read point coordinates in a Cartesian coordinate plane.


## Debriefing and Evaluation:

| Write 3 aspects you liked about this | 1. |
| :--- | :--- |
| activity: | 2. |
|  | 3. |
| Write 2 aspects that you have learned | 1. |
| Write 1 aspect for improvement | 2. |

## Introduction

Sometimes we find aspects related to Mathematics in television series or movies. In such cases, sometimes these Mathematical concepts are not given much importance, because they do not influence the story itself. However, there are a few cases in which they do.

Some examples include: "21" (USA, 2008), by Robert Luketic; "Proof" (USA, 2005), by John Madden; "A Beautiful Mind" (USA, 2001), by Ron Howard; "Enigma" (USA, 2001), by Michael Apted; "Pi" (USA, 1988), by Darren Aronofsky; "Good Will Hunting" (USA, 1997), by Gus Van Sant and "Cube" (Canada, 1997), by Vincenzo Natali.

In this tool, the movie "Kingdom of Heaven" (USA, 2005), by Ridley Scott, will be discussed and its mathematic concepts, such as the Cartesian coordinate systems, will be covered.

## Kingdom of Heaven

Kingdom of Heaven (2005) is a drama, action and war film, based on real events of the life of Balian, a young blacksmith who becomes the most honoured and heroic knight of the Kingdom of Jerusalem, protecting its people from all enemy threats. Balian, unhappy with the death of his wife and son, is sought by Godfrey of Ibelin - a renowned noble from the Kingdom of Jerusalem, who is dedicated to keep peace in the Holly Land. Godfrey confesses that he is his father and Balian will leave sadness behind to join him in is sacred mission. His father dies young and Balian inherits his land and title in Jerusalem, a city where Christians, Muslims and Jews struggle to coexist peacefully in the time between
 the second and the third crusade, during the $12^{\text {th }}$ century. Balian becomes the most honoured and heroic knight, protecting his people from the oppressors.

In 1187, Jerusalem is surrounded by Muslims, commanded by sultan Saladin, and even though the Christians were less, Balian creates a coordinate system to defend the city, which allows him to obtain more precision and optimize his war resources. With this strategy he manages to resist for a couple of days, finally reaching an agreement with the sultan, in which he compromised to give him Jerusalem in exchange for a free safe-conduct for all its inhabitants.

## Glossary

Christian: the one who professes Christianism (group of monotheist religions that believe in Christ's doctrine). The one who believes in Jesus Christ.

Crusade: military expedition led by the Christians during the Middle Ages with the objective of freeing the holy places, namely Jerusalem, from the Islamic power.

Jerusalem: current capital of Israel, located in the mountains of Judea between the Mediterranean and the Dead Sea, it is one of the most ancient cities in the world. It is considered to be holy by all three main monotheist religions: Judaism, Christianism and Islamism.

Jews: members of the ethnic and religious group original from the Tribes of Israel or the Hebrews from Ancient Orient.

Muslims: followers of the monotheist religion Islam, centred in the life and teachings of the prophet Mahomet.

Sultan: title given to the monarchs of certain Arabic or Islamic nations. The sultan is the one that holds the power in a certain region.

Safe-conduct: written license given to someone so that the person can walk freely anywhere, without being bothered.

Maths behind "Kingdom of Heaven"
As previously stated, Balian used coordinate systems to defend Jerusalem. In analytical geometry, there are various coordinate systems. The basic idea behind them is the representation of points in plane or in space, through the use of real number sets, known as coordinates. In this tool, we will focus on Cartesian coordinates plane.

## Cartesian coordinate plane

The basis of the union between geometry and algebra is the coordinate system which, in homage to the important French mathematician and philosopher René Descartes (1596-1650), is known as Cartesian coordinate or, simply, Cartesian plane.

The Cartesian coordinate, under focus, is formed by two perpendicular axes that intersect in a point - the origin of the system.

The horizontal axis is the $x \boldsymbol{x}$ or abscissa axis and the vertical axis is the $y \boldsymbol{y}$ or ordinate axis.

The positive part of the $\boldsymbol{x} \boldsymbol{x}$ axis stays at the right side of the origin and the positive part of the $y \boldsymbol{y}$ axis stays


Fig. 1 - René Descartes
(Source:https://commons.wikimedia.org/ wiki/Ren\%C3\%A9_Descartes\#/media/File: Frans_Hals_-
_Portret_van_Ren\%C3\%A9_Descartes.jpg) above the origin.

The axes divide the coordinate system in four quadrants, as is shown in figure 3 shows. Normally, in analytical geometry, we work with coordinate systems in which the measurement unit is the same for both axes, that is, monometric systems. If the axes are perpendicular, then we have an orthogonal coordinate system. Unless stated otherwise, we will use orthogonal monometric system.


Fig. 2 - Cartesian coordinate plane
(Source:https://commons.wikimedia.org/wiki/Twodimensional_coordinates?us elang=pt\#/media/File:Cartesian_coordinates_2D.svg)

## Points coordinates

To each point in the Cartesian plane corresponds an ordinated pair of numbers and vice-versa. This pair of numbers is known as the point coordinates.

Example: $\boldsymbol{P}(\mathbf{2}, \mathbf{3})$ abscissa $=2$ and ordinate $=3$. The points of the abscissa axis, $\boldsymbol{x} \boldsymbol{x}$, have null ordinate and the points of the ordinate axis, $y \boldsymbol{y}$, have null abscissa.

The points of the coordinate axes do not belong to any of the quadrants.


Fig. 3 - Points coordinates
(Source: https://www.desmos.com/)

The set of all ordinated real number pairs is designated as $\mathbb{R}^{2}$, meaning that $\mathbb{R}^{2}=\mathbb{R} \times \mathbb{R}=\{(\boldsymbol{x}, \boldsymbol{y}): \boldsymbol{x} \mathbf{e} \boldsymbol{y} \in \mathbb{R}\}$.

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## TASKS

## TASK 1

Observe the coordinate system and write the points coordinates presented.


Fig. 4 - Points coordinates
(Source: https://www.desmos.com/)

TASK2
Mark the following points in the Cartesian system:
$A(1,3) ; B(-2,2) ; C(0,-1) ; D(-3,-1) ; E(2,0) e$ $F(2,-3)$.


Fig. 5 - Cartesian system (Source: https://www.desmos.com/)

## TASK 3

Battleship with Cartesian coordinates. Activity for groups of three students (two players and one referee). Note: print the game boards available in the next pages and give them to the students.

## Game organization

1. Each player distributes their watercrafts through the board, marking the squares in which they will be anchored in the following way: aircraft carrier (five squares); four cannon ship (four squares); two three-cannon ships (three squares each) and four submarines (a square each), according to the shape seen in the figure.
2. Apart from the aircraft carrier, the watercrafts must occupy the

## 4 submarines <br> $\square$

3 two-cannon ships


2 three-cannon ships


1 four-cannon ship


1 aircraft carrier

(Adapted from:http://www.matematica.seed.pr.gov.br/modules/
conteudo/conteudo.php? conteudo=1320) squares in the extension of a line or a column. For example, a four cannon ship must occupy four squares in a line or column.
3. Two watercrafts cannot touch or overlap.
4. There should be at least one watercraft in each quadrant.
5. The referee should observe if the players are marking the points correctly in both boards (in the game board and in the adversary gunshots control board).

- The players should not reveal the location of the watercrafts to their opponent.
- The players decide who is the first to shoot.
- Each player, in his/her turn, will try to shoot the opponents watercraft. To do so, the player will indicate the $\boldsymbol{x}$ and $\boldsymbol{y}$ coordinates of that point (gunshot) in the Cartesian plane.
- The players should have in mind that $\boldsymbol{x}$ and $\boldsymbol{y}$ coordinates are ordinated pairs $(\boldsymbol{x}, \boldsymbol{y})$ and that the first number to be read should concern the $\boldsymbol{x}$ axis and the second the $\boldsymbol{y}$ axis.
- The opponent marks the corresponding point in his or her board and tells the attacker if she/he hit a watercraft or the water. If positive, the other player must tell which one it was. In case it sunk, that should also be informed. A watercraft sinks when all the squares that form it have been hit.
- So that the players can control the shots fired to their opponent, they should mark each point in the Cartesian coordinate plane correspondent to their adversary board.
- To hit a watercraft, hitting one of the vertices of the squares where it is anchored is enough.
- To sink a watercraft, the players need to hit at least two vertices of each square where the watercraft is anchored.
- If the player hits the target, she/he is entitled to play again successively until she/he hits the water or sinks all the watercrafts.
- If the player hits the water, she/he loses the turn. The same happens when a player marks the point incorrectly, in any of the boards. Those mistakes should be pointed out by the referee.
- The game ends when one of the players sinks all the opponents' watercrafts.

Adapted from:
http://www.matematica.seed.pr.gov.br/modules/conteudo/conteudo.php?conteudo=1320. Similar information may be found here: https://www.callmedrrob.com/?page_id=1805 (EN).

## Boards for the Battleship with Cartesian coordinates



Shots fired in my opponent's board


## LEARN MORE...

Kingdom of Heaven (2005) movie plo $\dagger$ https://www.imdb.com/title/tt0320661/?ref_=nv_sr_1 ${ }^{\text {ref_ }}=$ nv_sr_1 https://en.wikipedia.org/wiki/Kingdom of Heaven (film)

## Cartesian coordinate plane

https://www.khanacademy.org/math/basic-geo/basic-geo-coord-
plane/coordinate-plane-4-quad/v/the-coordinate-plane https://mathinsight.org/cartesian coordinates

Battleship (game)
https://en.wikipedia.org/wiki/Battleship (game)

Explore Maths with Desmos web application
https://www.desmos.com/

