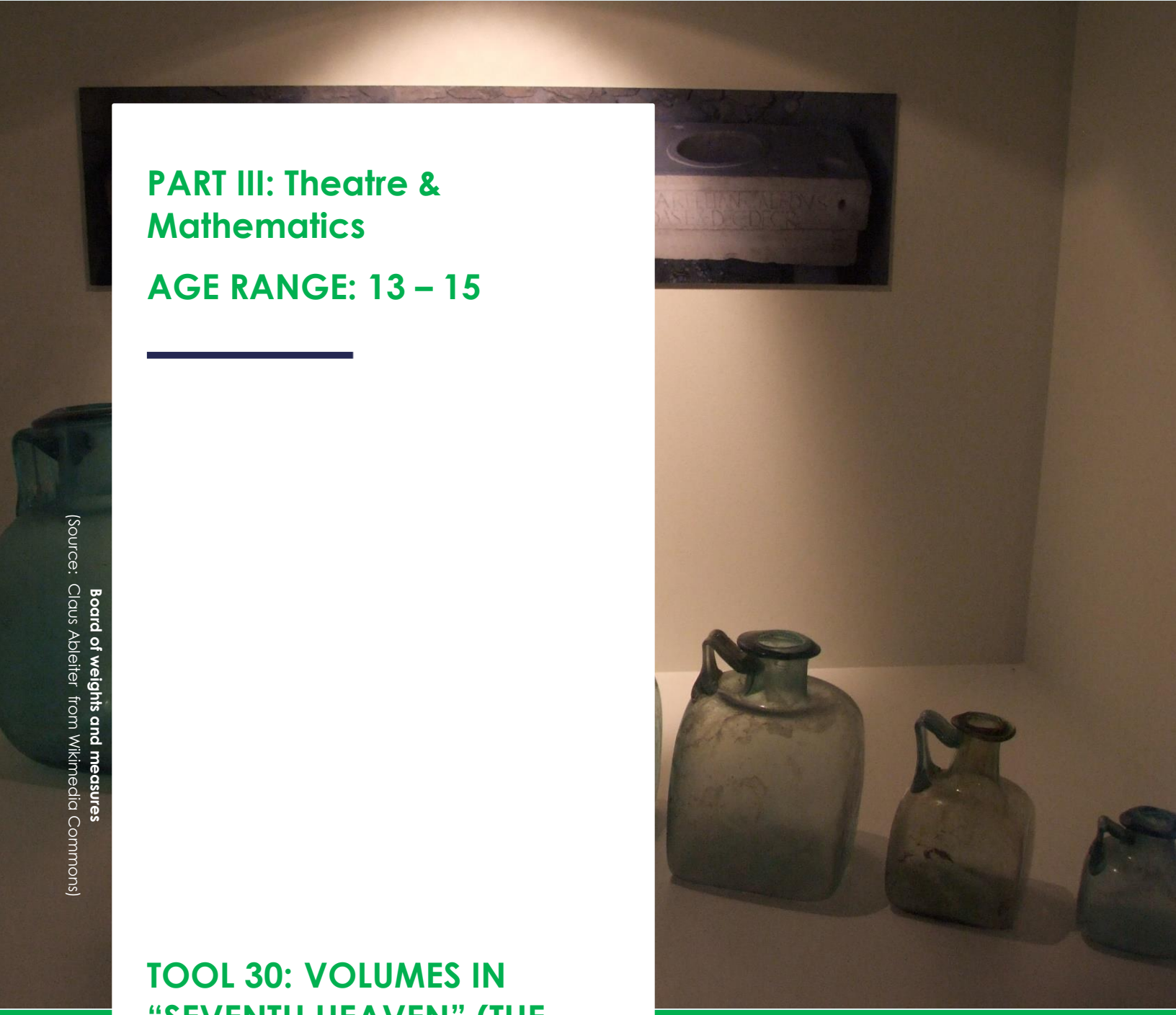

**Board of weights and measures**
(Source: Claus Ableiter from Wikimedia Commons)

# PART III: Theatre & Mathematics

## AGE RANGE: 13 – 15

## TOOL 30: VOLUMES IN "SEVENTH HEAVEN" (THE MAN WHO COUNTED, CHAPTER VIII)

SPEL – Sociedade Promotora de Estabelecimentos de Ensino

TheArtOf Maths

Co-funded by the Erasmus+ Programme of the European Union

# Educator's Guide

**Title:** Volumes in "Seventh Heaven" (The Man Who Counted, chapter VIII)

**Age range:** 13 – 15 years old

**Duration:** 3 hours

**Mathematical concepts:** Volumes

**Artistic concepts:** Theatrical Performance

**General objectives:** Role-play an adapted scene of "Seventh Heaven" from the book "the Man Who Counted"; Calculate volumes of geometric solids

**Instructions and Methodologies:** Together with the adapted script and the explanation of the "Seventh Heaven", it is important for the students to have some examples of the calculation of volumes before solving the exercises

**Resources:** Computer with an internet connection; Scientific calculator

**Tips for the educator:** Start by giving several examples of the calculation of the volumes of geometric solids, with an increasing level of difficulty, in order to teach them how to solve the exercises by themselves. Provide the students with the adapted script of the "Seventh Heaven", in order to help them rehearse that scene



**Learning Outcomes and Competences:**

At the end of this module, the student will be able to:

- Understand the solution presented in the "Seventh Heaven";
- Role-play an adapted scene of the "Seventh Heaven";
- Calculate volumes of geometric solids

**Debriefing and Evaluation:**

| | |
|---|---|
| Write 3 aspects you liked about this activity: | 1.<br>2.<br>3. |
| Write 2 aspects that you have learned | 1.<br>2. |
| Write 1 aspect for improvement | 1. |

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# Introduction

Over the course of history, Mathematics found the answers to several problems that emerged. There are many reports of those solutions and there is no doubt that Mathematics is extremely relevant in problem-solving and had an important role in the evolution of diverse civilisations throughout history.

Who has never heard a tale or read a book in which Mathematics ended up by being involved in problem or enigma solving?

This module will address a mathematical problem that appears in a book full of similar situations. The book, "The Man Who Counted", written in 1938, by Malba Tahan (pseudonym of the professor and writer Júlio César de Mello e Souza), tells the story of Beremiz Samir, a Persian traveller with a special talent for Mathematics, who uses and implements logical thinking and other mathematical concepts to solve daily problems in each place that he visits.



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# Volumes in "Seventh Heaven"

The book "The Man Who Counted", by Malba Tahan, pseudonym of the Brazilian professor and writer Júlio César de Mello e Souza, recreates a series of mathematical problems and puzzles involving arithmetic, algebra, geometry, and other fields of Mathematics.

The book tells the story of Hanak Tade Maia, a man that is travelling from Samarra to Baghdad. On the way, he meets Beremiz Samir, a Persian man with remarkable mathematical skills, and invites the man to join him on his trip. For Hanak, it was certain that a man with such mathematical abilities would find a profitable job in Baghdad.

One of the problems presented in the book is the division of twenty-one wine casks by three sheep rearers.



In the book, Beremiz and Hanak have an encounter with the Sheik Salem Nasair and his friends, the sheep rearers, and the Sheik asks Beremiz to solve their problem, regarding the division of the twenty-one wine casks.

According to the book, the Sheik says to Beremiz:
"– Here are my three friends. They are sheep rearers from Damascus. They are facing one of the strangest problems I have come across. It is this: as payment for a small flock of sheep, they received here in Baghdad, a quantity of excellent wine, in twenty-one identical casks: seven full, seven half-full, and seven empty. They want to divide so that each receives the same number of casks and the same quantity of wine. Dividing up the casks is easy – each would receive seven. The difficulty, as I understand, is in dividing the wine without opening them, leaving them just as they are. Now, is it possible to find a satisfactory answer to this problem?

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Beremiz, after pondering for two or three minutes, replied:

– The division of the twenty-one casks, O Sheik, can be done without much complication. I am going to suggest the simplest possible solution. The first will receive three full casks, one half-full, and three empty, for a total of seven casks. The second will receive two full casks, three half-full, and two empty, for a total of seven casks.

The third will also receive seven casks in the same arrangement. According to my division, each party will acquire seven casks and equal quantity of wine."[1]

This solution proposed by Beremiz solved the problem since each friend would receive seven wine casks and **the same amount of wine**.

Beremiz wanted to prove that the amount of wine was the same, and managed to make it by attributing to the portion of the wine in each full wine cask the number 2, and to the portion of the wine in each half-full wine cask the number 1.

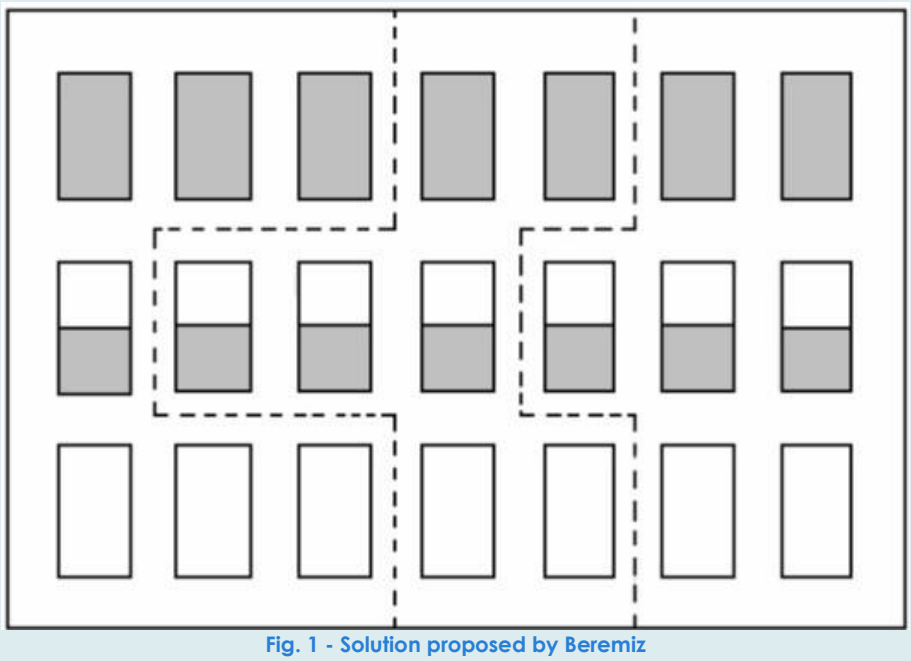

**Fig. 1 - Solution proposed by Beremiz**
(Source: Taham, M. (n.d.). O Homem que calculava. Retrieved from: http://josenorberto.com.br/o_homem_que_calculava.pdf (16.07.2019))



[1] Tahan, M. (n.d.). The Man Who Counted – A Collection of Mathematical Adventures. Retrieved from: https://sparthasarathy.com/ebooks/themanwhocounted.pdf (16.07.2019).

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According to the book, Beremiz declared:

"- According to the division, the first partner will receive 2 + 2 + 2 + 1, a total of seven units, and each of the others will receive 2 + 2 + 1 + 1 + 1, also adding up to seven. This proves that my suggested division is exact and just. Although the problem appears complicated, its numerical resolution presents no difficulty.

His solution was received with much enthusiasm, not just by the Sheik, but also by the three men of Damascus."[1]

Another solution could be: one of the sheep rearers would receive one full, five half-full, and one empty wine cask. The other two sheep rearers would receive three full, one half-full, and three empty wine casks.

In order to solve this problem, Beremiz resorted to arithmetic. However, if we only consider the division of the wine, we have, as well, a matter of volumes, which will be addressed in this module.



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# Glossary

**Bagdad:** Ancient centre of the Islamic world, and the current capital of Iraq.

**Damascus:** Capital and country's largest city of the Syrian Arab Republic.

**Persian:** Native person from Persia (a word used by the Greeks of the Classical Era and by the Western to refer to all the Iran plain.

**Samarra:** City in Iraq, located in the western side of the Tigris, in the province of Saladino, 125 km north of Bagdad.

**Sheik:** A honorific title in the Arabic language that designates the ruler of a tribe or a royal family member.



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# The Math behind "Seventh Heaven"

As mentioned previously, if we only consider the division of the wine, we will have a problem of volumes, so this theme will be addressed ahead.

## Volumes

The **volume of a geometric solid** is the space occupied by it. The most used units of volume most used are the cubic metre ($m^3$), the cubic decimetre ($dm^3$), and the cubic centimetre ($cm^3$). One cubic metre of volume corresponds to one thousand litres of capacity and one cubic decimetre of volume corresponds to one litre of capacity.

According to the International System of Units (SI), the cubic metre is the standard unit for the volume measurements.

We will try to simplify the formulas for the calculation of the volumes of the solids by making a division into "solids with two bases", "solids with one base", and "solids without bases".



## The volumes of "solids with two bases"

The volumes of solids with two bases (e.g. cube, parallelepiped, triangular or pentagonal prisms, cylinder) are always the same as the result of the multiplication of the base area by the height.

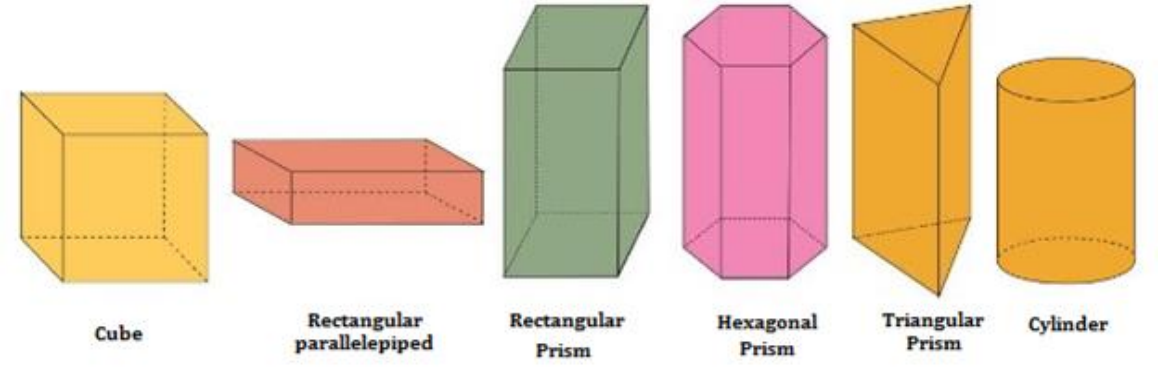


**Fig. 2 – Solids with two bases**
(Source: Image adapted from Ação Educativa (2015, July 3). Mundo em construção – 8º ano. Retrieved from: https://issuu.com/acaoeducativa/docs/8___ano_-_mundo_em_constru____o_alu/404 (16.07.2019))

$\mathbf{V = A_b \times h}$, where $A_b$ is the base area and $h$ is the height

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Example: Calculate the volume of the following rectangular prism:

$$\mathbf{V = A_b \times h = (1 \times 8) \times 5 = 40\ cm^3}$$

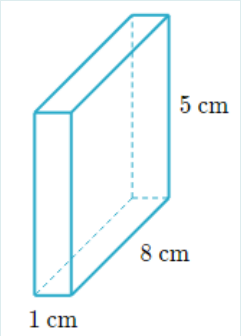


**Fig. 3 – Rectangular Prism**
(Source:https://pt.khanacademy.org/math/basic-geo/basic-geo-volume-sa/volume-rect-prism/e/volume_1)

## The volumes of "solids with one base"

The volumes of solids with only one base (e.g. pyramid, cone) are always the same as the result of the multiplication of 1/3 of the base area by the height.

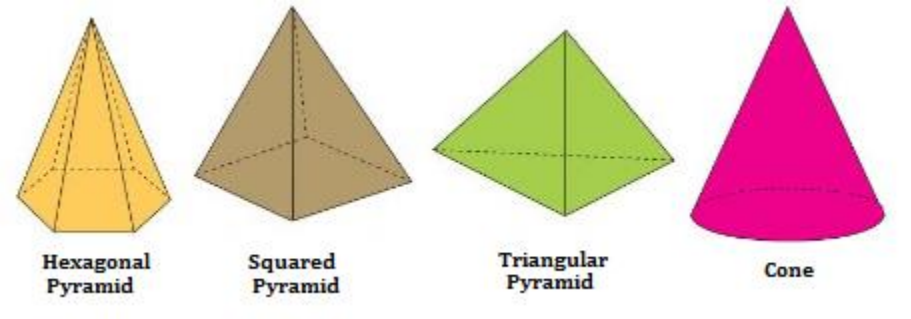


**Fig. 4 – Solids with one base**
(Source: Image adapted from Ação Educativa (2015, July 3). Mundo em construção – 8º ano. Retrieved from: https://issuu.com/acaoeducativa/docs/8___ano_-_mundo_em_constru____o_alu/404 (16.07.2019)

$\mathbf{V = \frac{1}{3}A_b \times h}$, where $\boldsymbol{A_b}$ is the base area and $\boldsymbol{h}$ is the height

Example: Calculate the volume of the following cone, considering the dimensions in centimetres.

$$\mathbf{V = \frac{1}{3}A_b \times h = \frac{1}{3}(\pi \times 2^2) \times 9 = \frac{1}{3}(4\pi) \times 9 = \frac{36\pi}{3} = 12\pi\ cm^3}$$

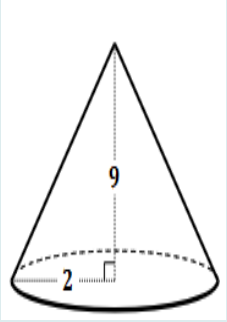


**Fig. 5 – Cone**
(https://www.khanacademy.org/math/basic-geo/basic-geo-volume-sa/volume-cones/e/volume-of-cones)



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## The volume of a "solid without a base"

The volume of a solid without a base (i.e. sphere) is the same as the result of the multiplication of 4/3 by $\pi r^3$.

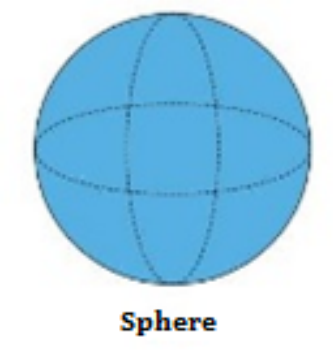


**Fig. 6 - Sphere**
(Source: Ação Educativa (2015, July 3). Mundo em construção – 8º ano. Retrieved from: https://issuu.com/acaoeducativa/docs/8___ano_-_mundo_em_constru____o_alu/404 (16.07.2019))

$\mathbf{V_{Sphere}} = \frac{4}{3} \times \pi \mathbf{r^3}$, where $r$ is the radius of the sphere

Example: Calculate the volume of the following sphere, considering the dimensions in centimetres.

$$\mathbf{V} = \frac{4}{3} \times \pi \mathbf{r^3} = \frac{4}{3} \times \pi \times \mathbf{3^3} = \frac{4}{3} \times \pi \times \mathbf{27} = \frac{\mathbf{108\pi}}{\mathbf{3}} = \mathbf{36\pi\ cm^3}$$

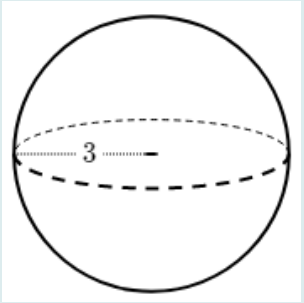


**Fig. 7 - Sphere**
(Source: https://pt.khanacademy.org/math/basic-geo/basic-geo-volume-sa/volume-cones/e/volume-of-spheres)



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# TASKS

## TASK 1

**1.** Role-play the adapted scene from the book "The Man Who Counted" regarding the chapter VIII, "Seventh Heaven", using the script attached.

## TASK 2

**2.** The cube in the figure below has an edge of 6 cm and the shaded pyramid has of height one-third of the cube's height.

**2.1.** Calculate the volume of the cube.

**2.2.** Calculate the volume of the pyramid.

**2.3.** Calculate the volume of the cube that is not filled by the pyramid.

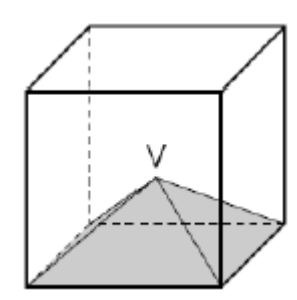


**Fig. 8 - Cube and Pyramid**
(Source: Neves, M. A, Pereira, A., Leite, A., Guerreiro, L., & Silva, M. C. (2006). Matemática A1 – Ensino Profissional: Geometria. Porto: Porto Editora.)



## TASK 3

**3.** In a cylindrical vessel with a diameter of 20 cm, a determined amount of water was stored. Afterwards, a metallic sphere with a diameter of 6 cm was inserted in the vessel. It was possible to observe that the level of water remained tangent to the sphere.

What was the volume of water that was inserted in the vessel?

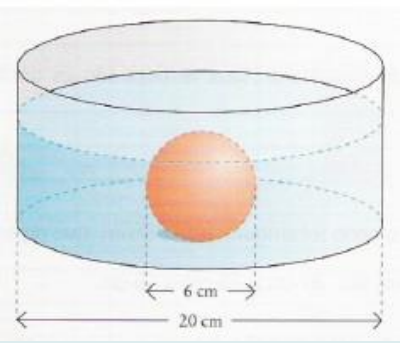


**Fig. 9 – Cylindrical vessel and sphere**
(Source: Neves, M. A, Pereira, A., Leite, A., Guerreiro, L., & Silva, M. C. (2006). Matemática A1 – Ensino Profissional: Geometria. Porto: Porto Editora)

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# LEARN MORE...

The Man Who Counted – A Collection of Mathematical Adventures

https://sparthasarathy.com/ebooks/themanwhocounted.pdf

Solid Geometry

https://www.onlinemathlearning.com/solid-geometry.html

https://www.mathsisfun.com/geometry/solid-geometry.html

What Are Solid Geometric Shapes or Bodies?

https://www.smartickmethod.com/blog/math/geometry/solid-geometric-shapes/

Basic Geometry – Volume and Surface Area

https://www.khanacademy.org/math/basic-geo/basic-geo-volume-sa

The Volume of a Cube

https://www.onlinemathlearning.com/volume-of-a-cube.html

The Volume of Prisms

https://www.onlinemathlearning.com/volume-prism-1.html

The Volume of Rectangular Prisms

https://www.khanacademy.org/math/geometry-home/geometry-volume-surface-area#geometry-volume-rect-prism

https://www.onlinemathlearning.com/volume-rectangular-prism.html



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The Volume of Cones, Cylinders, and Spheres

https://www.khanacademy.org/math/geometry-home/geometry-volume-surface-area#geometry-volume-cones

The Volume of Cylinders

https://www.onlinemathlearning.com/volume-of-a-cylinder.html

The Volume of a Sphere

https://www.onlinemathlearning.com/volume-of-a-sphere.html

The Volume of a Cone

https://www.onlinemathlearning.com/volume-cone.html



The Volume of a Pyramid

https://www.onlinemathlearning.com/volume-of-a-pyramid.html

Exploring Nets of Geometric Solids

https://www.geogebra.org/m/n6EjQDw8

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# "SEVENTH HEAVEN" PLAY

(adapted from the book "The Man Who Counted")

**Script of the adaptation of the "Seventh Heaven" chapter from the book "The Man Who Counted":**

Characters presented in the scene:

- Hanak Tade Maia;
- Beremiz Samir;
- Sheik Salem Nasair;
- 1st Muslim sheep rearer;
- 2nd Muslim sheep rearer;
- 3rd Muslim sheep rearer.

Materials (minimum):

- 21 vessels of equal sizes;
- 1 bottle of water.



[In the scene, there are 3 Muslim sheep rearers and the Sheik Salem Nasair. They are seated, fraternizing. Hanak Tade Maia and Beremiz Samir approach them.]

**Sheik Salem Nasair** (raising his arms to the sky)**:** Here he is, the esteemed master calculator! Welcome, my friend!

**Beremiz Samir:** How are you, my friend?

[The Sheik and Beremiz hug each other].

**Beremiz Samir:** This is my friend Hanak. He invited me to join him on his journey.

**Hanak Tade Maia:** Hello, Sheik! Nice to meet you.

**Sheik Salem Nasair:** Hello, Hanak! If you are a friend of Beremiz, you are also my friend! Those are my three friends from Damascus.

[The Muslims stand up in order to greet the travellers.]

**1st Muslim:** Marhabaan almusafirin!

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**2nd Muslim:** "Marhabaan!

**3rd Muslim:** "Marhabaan! Kayf halikm?

[Beremiz and Hanak greet the three men.]

**Beremiz Samir:** So, Nasair, how are you?

**Sheik Salem Nasair:** Well, Beremiz, I am very well, thank you, but my friends here are facing one of the strangest problems I have come across. Yet, maybe you can help them solving it, due to your extraordinary mathematical skills!

**Beremiz Samir:** What seems to be the problem?

**Sheik Salem Nasair:** My three friends are sheep rearers from Damascus and, as payment for a small flock of sheep, they received here, in Baghdad, a quantity of excellent wine, in twenty-one identical casks. However, seven are full, the other seven are half-full, and the last seven wine casks are empty.

**Beremiz Samir:** I see.

**Sheik Salem Nasair:** In this sense, they want to divide the wine so that each one receives the same number of casks and the same amount of wine.



**1st Muslim:** Dividing up the wine casks is easy, as each one would receive seven.

[The other two men agree with the statement and make gestures with their heads in order to show their agreement.]

**Sheik Salem Nasair:** The difficulty, as I understand, relies on dividing the wine without opening the wine casks, leaving them just as they are now.

**Beremiz Samir:** Yes, I understand.

**Hanak Tade Maia:** Beremiz, do you think that it is possible to find a satisfactory answer to this problem?

**2nd Muslim:** Traveller, do you think you can help us with our strange problem?

**Beremiz Samir:** I think I can. Let me think about on this problem for a little.

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[Beremiz lowers his head and moves around a little, back and forth, and reflects over a few seconds on the problem. After a few seconds, Beremiz raises his head and addresses the people that are with him.]

**Beremiz Samir:** The division of the twenty-one wine casks can be done without much complication. I am going to suggest the simplest possible solution.

**1st Muslim:** Please, tell us what we should do.

[Beremiz turns to the first Muslim man.]

**Beremiz Samir:** You will receive three full wine casks, one half-full, and 3 empty, for a total of seven casks.

[Then, Beremiz turns to the second Muslim man.]

**Beremiz Samir:** You will receive two full wine casks, three half-full, and 2 empty, for a total of 7 casks.

[Finally, Beremiz turns to the third Muslim man.]

**Beremiz Samir:** You will receive two full wine casks, three half-full, and two empty, as well.



[After that, Beremiz addresses the Sheik Nasair.]

**Beremiz Samir:** According to my division, each party will acquire seven casks and an equal quantity of wine.

**Sheik Salem Nasair:** I am not sure if I understood your division.

**Hanak Tade Maia:** Me neither, Beremiz.

**Beremiz Samir:** No problem, I will explain my reasoning behind this division. Let us say that a full cask of wine holds two portions, and a half-full cask of wine holds one portion, right?"

**Sheik Salem Nasair:** Right.

**Hanak Tade Maia:** Right.

**Beremiz Samir:** In this way, according to the division, the first man will receive 2 + 2 + 2 + 1, a total of seven units. Are you following?

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**Sheik Salem Nasair:** Yes.

**Hanak Tade Maia:** Yes, I am following.

**Beremiz Samir:** Consequently, each one of the remaining men will receive 2 + 2 + 1 + 1 + 1, also adding up to seven.

**Sheik Salem Nasair and Hanak:** Oh!! Now, I understood!!

**Beremiz Samir:** This proves that my suggested division is exact and just. Although the problem appears complicated, its numerical resolution presents no difficulty.

[The solution of Beremiz was received with much enthusiasm by all, particularly the three men of Damascus.]

**1st Muslim:** By Allah! This calculator is amazing! In a moment, he cleared up a problem that seemed most difficult for us.

**2nd and 3rd Muslims:** By Allah!

**1st Muslim:** Shukraan jazilaan.



**2nd and 3rd Muslims:** Shukraan.

**Beremiz Samir:** You are welcome! I am happy that I could help!

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