

PART III: Theatre & Mathematics

AGE RANGE: 13-15

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"Multicolored Abacus"

TOOL 28: BASIC ARITHMETIC IN "BEASTS OF BURDEN" (THE MAN WHO COUNTED, CHAPTER III)

SPEL – Sociedade Promotora de
Estabelecimentos de Ensino



Co-funded by the
Erasmus+ Programme
of the European Union

Educator's Guide

Title: Basic Arithmetic in “Beasts of Burden” (The Man Who Counted, Chapter III)

Age range: 13 – 15 years old

Duration: 2 hours

Mathematical concepts: Decimal, Fraction, Order of operation, Percentages, Ratio and Proportions

Artistic concepts: Arithmetic

General objectives: To use deductive reasoning when solving mathematic problems and develop the confidence to use mathematics to analyze and solve problems in school and real-life contexts

Instructions and Methodologies: When dramatizing, have the students go nearby the whiteboard so that they can write down the divisions or draw camels in order for students to have a better understanding of the divisions being made

Resources: Pen, whiteboard

Tips for the educator: Before the dramatization, promote a rehearsal and decide the positions each character is going to take.

Learning Outcomes and Competences:

At the end of this lesson the student will be able to:

- o add, subtract, multiply and divide fractions; - convert fractions into decimals and decimals into fractions; - multiply and divide decimals by multiples of 10; change a fraction to a percentage; - find a percentage of a number; increase/decrease a number by a percentage; - use ratio and division in proportion; - understand the order of arithmetic operations; and - use basic calculator functions.

Debriefing and Evaluation:

Write 3 aspects you liked about this activity:	1. 2. 3.
Write 2 aspects that you have learned	1. 2.
Write 1 aspect for improvement	1.

Introduction

The first records of the use of arithmetic were found in historical monuments of Babylon and Ancient Egypt that date around 2,500 years BC. However, it was later in Greece, around the 6th century BC, that ancient mathematicians, especially the Pythagoreans, pushed arithmetic forward by trying to match all regularities of the world to numbers. Around the 17th century, Astronomy and Mechanics brought forward new challenges regarding calculation, which, as a result, gave impulse to the further development of arithmetic.

The common objective of arithmetic is to develop the skill of being able to do basic mental calculus, without having to rely on paper, pencil, calculators, or any other auxiliary device. In fact, people face daily situations in which they have to rely on arithmetic without even noticing, be it paying for a coffee and calculating if the change is correct, or even checking how much time is left until the beginning of a class...

Basic Arithmetic in “Beasts of Burden”

When studying, you will come across topics that involve the mathematics of numbers. What these topics all have in common is that they are solved under the operations of addition, subtraction, division and multiplication.

In the book “The man who counted”, by Malba Tahan, pseudonym of the Brazilian writer Júlio César de Mello e Souza, a series of mathematical problems and puzzles involving arithmetic are recreated.

The book, which is written in the style of the Arabian Nights, tells the story of Hanak, a man who is traveling from Samarra to Baghdad. On the way, he meets Beremizer Samir, a man with remarkable mathematical skills, who is invited to join the trip. For Hanak, it is certain that a man with such mathematical abilities will find a profitable job in Baghdad.

In chapter III, Beasts of Burden, in one of his most famous problems, he witnesses a heated dispute between three brothers who are having a hard time dividing 35 camels left by their father. The three brothers' father has left them 35 camels referring that $\frac{1}{2}$ (17,5) of the camels should belong to the oldest brother, $\frac{1}{3}$ (11,6) to the middle one and $\frac{1}{9}$ (3,8) to the youngest.

Beremizer offers his help to solve the dilemma by adding Hanak's camels to the count. At first, Hanak resists because their journey to Baghdad without a camel is almost impossible. However he gives in and eventually releases the camel. The three brothers are very satisfied with this solution, since another camel is added.

Now, with 36 camels, Beremizer can fairly divide the camels respecting the instructions left by the brothers' father. That is, $\frac{1}{2}$ of 36 is 18 and is the number of camels the oldest brother will receive; $\frac{1}{3}$ of 36 is 12, corresponding to the camels the middle brother is entitled to and, finally, $\frac{1}{9}$ of 36 is 4, the number of inherited camels by the youngest brother.

However, $18+12+4 = 34$, which means that 2 camels are left out. The brothers were quite pleased by the solution to their problem found by Beremiz, that they accept his suggestion of giving back to his friend Hanak 1 camel, and keeping the other to himself.

Glossary

Arabian Nights – Originally “One Thousand and One Nights”, refers to a collection of folk tales from the Middle East and South of Asia compiled in Arabic language during the Islamic Golden Age (8th century to 14th century).

Arithmetic - the science of numbers and a branch of Mathematics that involves any operation that is within the number's realm.

Bagdad: Ancient centre of the Islamic world, and the current capital of Iraq.

Samarra: City in Iraq, located in the western side of the Tigris, in the province of Saladino, 125 km north of Bagdad.

The Math behind “Beasts of Burden”

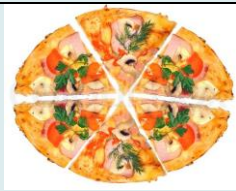
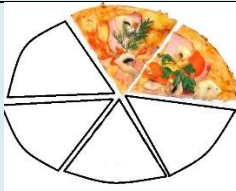
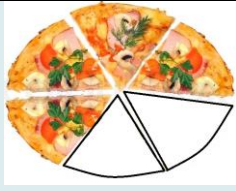
By appealing to his knowledge of arithmetic, such as fractions, decimal numbers, ratio and proportions, percentages and order of operation, Beremiz is able to solve many dilemmas brought forward throughout his journey.

Below, an introduction of the aforementioned concepts will be covered.

1. Fractions

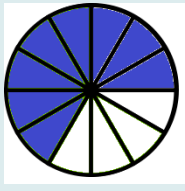
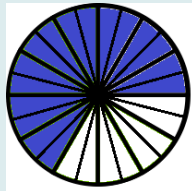
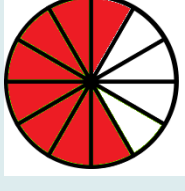
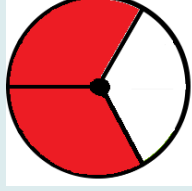
A fraction equals to a **part** of a **whole**, in which the **part** is represented as the **numerator**, the top symbol or number, and the **whole** as the **denominator**, the bottom symbol or number. They are displayed the following way:

$\frac{\text{Numerator}}{\text{Denominator}}$

<p>If you cut a pizza into 6 different slices, there will be 6 slices out of the whole slices.</p>	$\frac{6}{6}$	
<p>You are feeling hungry, so you take 2 slices for you. You will have taken 2 slices out of 6.</p>	$\frac{2}{6}$	
<p>Since you took 2 slices, there will be 4 slices left out of the 6 that there were.</p>	$\frac{4}{6}$	

One particularity that fractions have is that they can be amplified (multiplied) or simplified (divided), while still keeping the same values. These are called **equivalent fractions**.

Consider the following fraction: $\frac{8}{12}$

<p>Amplification: multiply both the numerator and the denominator by the same number;</p>	$\frac{8}{12} = \frac{24}{32}$ <p style="text-align: center;">x3</p>		<p style="text-align: center;">=</p>	
<p>Simplification: divide both the numerator and the denominator by the same number;</p>	$\frac{8}{12} = \frac{2}{3}$ <p style="text-align: center;">:4</p>		<p style="text-align: center;">=</p>	

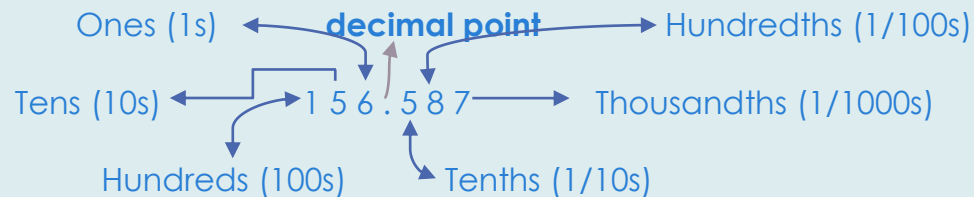
Fractions can also be divided and multiplied, as well as added or subtracted:

<p>Multiplying:</p> <ol style="list-style-type: none"> 1) Multiply the numerators; 2) Multiply the denominators; 3) Simplify (if possible). 	$\frac{8}{12} \times \frac{2}{3} = \frac{16}{36} = \frac{4}{9}$ <p style="text-align: center;">:4</p>
<p>Dividing:</p> <ol style="list-style-type: none"> 1) Get the reciprocal by inverting one of the fractions; 2) Follow the multiplication steps 1 to 3; 	$\frac{8}{12} \div \frac{5}{3} = \frac{8}{12} \times \frac{3}{5} = \frac{24}{60} = \frac{2}{5}$ <p style="text-align: center;">:12</p>
<p>Adding and subtracting:</p> <ol style="list-style-type: none"> 1) Find a common denominator; 2) Amplify the fractions; 3) Add or subtract the numerator; 4) Simplify (if possible). 	$\frac{8}{12} + \frac{2}{5} = \frac{8 \times 5}{12 \times 5} + \frac{2 \times 12}{5 \times 12}$ $\frac{40}{60} + \frac{24}{60} = \frac{64}{60} = \frac{16}{15}$ <p style="text-align: center;">:4</p>

2. Decimals

The word “decimal” derives from the Greek word “decima”, which means “tenth part”. When we write decimal numbers, the position of each number is very important.

Let us analyze the following decimal number:



The **decimal point** of a number is its most important part. As we move to the right of the decimal point, each position grows 10 times smaller; as we move to the left, each position grows 10 times bigger.

Converting decimals to fractions		Converting fractions into decimals	
$0,4 = \frac{4}{10}$	$0,56 = \frac{56}{100}$	$\frac{2}{7} = 2 : 7 = 0,28$	$\frac{12}{6} = 12 : 6 = 2,0$
$8,75 = 8\frac{75}{100}$	$2,758 = 2\frac{758}{1000}$	$\frac{52}{100} = 52 : 100 = 0,52$	$\frac{156}{1000} = 156 : 1000 = 0,156$
Multiplying a decimal by a multiple of 10		Dividing a decimal by a multiple of 10	
$15,7 \times 10 = 157$	$32,458 \times 100 = 3245,8$	$15,7 : 10 = 1,57$	$32,458 : 100 = 0,32458$

3. Ratios and proportions

When someone talks about the speed of a car, the expression “miles per hour” is often used. This expression is a type of ratio, which, fundamentally, compares miles to hours. For instance, if a camel is speeding 30 miles per hour, it means that, if the

camel maintains the same exact speed, for every hour, the camel will have ridden 30 miles.

Ratios are read as “the ratio of x to y” and is usually written the following way:

$$160:1$$

On the other hand, proportions are equations that state an equivalence between two ratios. They can, likewise, be displayed the way ratios are (using a colon between the two numbers) or as fractions. The following proportion is read as “ten is to twenty-five as two is to five”:

$$\frac{10}{25} = \frac{2}{5}$$

To find out if two ratios are equal, that is, if it is a true proportion, we multiply the outer terms (called extremes) and the middle terms (called means) and check if the product is the same. In the displayed example, the extremes are 10 and 5 and the means are the 25 and the 2. So, $10 \times 5 = 50$ as $25 \times 2 = 50$, proves it is a true proportion.

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4. Percentages

Percentages are usually represented by the percentage sign “%” and are used to express a proportional part of a whole. In other words, a percentage number is a number that represents a fraction out of 100, i.e., a number with 100 as the denominator. Hence it can be written as a fraction, a ratio or a decimal.

For instance 32% (read as thirty-two percent), which means 32 per 100, can also be written and has the same value as 0,32 or $\frac{32}{100}$ or 32:100.

The easiest method to obtain a percentage out of a fraction is simply by dividing the numerator by the denominator and then multiply the result times 100.

5. Order of operation

Long time ago, mathematicians decided on which should be the right order to solve an operation. This decision, adopted by many, facilitated the communication between mathematicians. Eventually, it became a general rule to follow.

$\frac{50}{100} = 0,5; 0,5 \times 100 = 50\%$	$\frac{5}{9} = 5; 5 \times 100 = 55\%$
$\frac{85}{231} = 0,36; 0,36 \times 100 = 36\%$	$\frac{59}{79} = 0,74; 0,74 \times 100 = 74\%$

When making a calculation, you always have to start from **left to right**, always prioritizing the following:

1) Parenthesis	$18 \times (5 + 3) = 18 \times 7 = 126$	Correct
	$18 \times 5 + 3 = 90 + 3 = 93$	Wrong
2) Exponents (Powers, Roots)	$3 \times 10^2 = 3 \times 100 = 300$	Correct
	$3 \times 10^2 = 30^2 = 900$	Wrong
3) Multiplication and Division	$3 + 6 \times 2 = 3 + 12 = 15$	Correct
	$3 + 6 \times 2 = 8 \times 2 = 16$	Wrong
	$40 - 10 : 5 = 40 - 2 = 38$	Correct
	$40 - 10 : 5 = 30 : 5 = 6$	Wrong
4) Left to right	$10 : 5 \times 3 + 2 = 2 \times 3 + 2 = 6 + 2 = 8$	Correct
	$10 : 5 \times 3 + 2 = 10 : 15 + 2 = 0,66 + 2 = 2,66$	Wrong

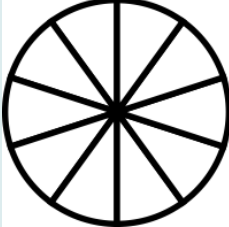
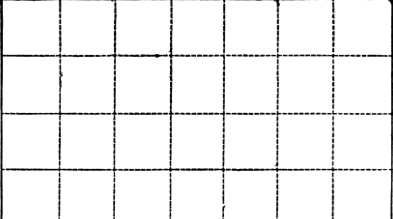
This means that if you do not follow the right order you will get a wrong result.

TASKS

TASK 1



Without relying on any resources, choose the correct option:

<p>1) John bought a box of 8 eggs. He boiled 3 of them. Which fraction corresponds to the number of eggs left in the box?</p>	<p>a. $\frac{3}{8}$</p>	<p>b. $\frac{5}{8}$</p>	<p>c. $\frac{8}{3}$</p>
<p>2) Mary is playing with a deck of 40 cards. She divides the cards equally between her and her 7 friends. Which fraction corresponds to the number of cards that each friend has?</p>	<p>a. $\frac{5}{40}$</p>	<p>b. $\frac{6}{40}$</p>	<p>c. $\frac{10}{40}$</p>
<p>3) Tobias baked a cake for his birthday. He called 5 of his friends over his house to celebrate with him. After he blew the candles, he cut the cake in equal slices. Everyone ate 2 slices, before the cake was long gone. Which fraction corresponds to the part of the whole cake that each person ate?</p>	<p>a. $\frac{2}{10}$</p>	<p>b. $\frac{1}{6}$</p>	<p>c. $\frac{10}{12}$</p>
<p>4) Colour the figures according to each fraction:</p>			
<p>4.1) $\frac{1}{5}$</p>			
<p>4.2) $\frac{1}{7}$</p>			

5) Convert the fractions into decimals and vice versa:			
5.1) $\frac{50}{100}$	a. 5	b. 0,5	c. 0,05
5.2) $\frac{3}{1000}$	a. 0,3	b. 0,03	c. 0,003
5.3) $\frac{60}{90}$	a. 0,33	b. 0,66	c. 1,00
5.4) 0,45	a. $\frac{45}{10}$	b. $\frac{45}{100}$	c. $\frac{45}{1000}$
5.5) 0,78	a. $\frac{78}{1}$	b. $\frac{78}{10}$	c. $\frac{780}{1000}$
5.6) 0,055	a. $\frac{1,55}{100}$	b. $\frac{55}{100}$	c. $\frac{55}{1000}$
6) Find out the right ratios and proportions:			
6.1) A water well is draining 5 liters of water per hour.	a. 2:10	b. 10:2	c. 5:2
6.2) A man is running at 6 km/h.	a. 1:6	b. 6:6	c. 30:5
6.3) Twenty-eight is to fifty as:	a. $\frac{2,8}{5}$	b. $\frac{28}{25}$	c. $\frac{280}{100}$
6.4) Two-hundred is to one-thousand as:	a. $\frac{50}{250}$	b. $\frac{40}{100}$	c. $\frac{2000}{5000}$
7) A bucket takes 16 apples to be full. John's bucket has 6 apples. What percentage corresponds to the space that is left in the bucket?	a. 62,5%	b. 40%	c. 50%

8) Mary takes 10 sips to drink all the water from her bottle. She has taken 2 sips already. What percentage of water has she drank?	a. 10%	b. 20%	c. 80%
9) $10 \times (3 + 5) =$	a. 35	b. 80	c. 18
10) $30 + 8 (2 \times 2) =$	a. 76	b. 152	c. 62
11) $(16+3) - 5^2 + (2 \times 2) =$	a. 33	b. -2	c. 18

TASK 2



Propose students to replicate the “Beasts of Burden” chapter as a theatre play. To do so, follow the script displayed below.

“BEASTS OF BURDEN” PLAY

(adapted from the book “The Man Who Counted”)

Cast of Characters:

Hanak (student 1) – a lone person, traveler and wise man;

Beremizer Samir (student 2) – The protagonist; the companion of Hanak, a young man with amazing mathematical skills;

Youngest brother (student 3);

Middle brother (student 4);

Older brother (student 5).

[Students in the classroom (excluding student 1 and 2), gather around the three brothers who are having a heated dispute. Hanak and Beremizer are on the opposite side of the room, slowly walking towards the crowd]

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YOUNGEST BROTHER: It cannot be! This is totally unfair

MIDDLE BROTHER: What do you mean?! This is robbery!

OLDEST BROTHER: This is the fairest division there could be! Take it or leave it!

YOUNGEST BROTHER: Are you robbing me because I'm the youngest? I'm the youngest but I'm certainly not the dumbest! I want a fair share! I do not agree with this!

MIDDLE BROTHER [Looking at the Oldest Brother while pointing at the Youngest Brother]: He is right! This is outrageous!

[While some students are trying to ease the situation by appealing the brothers to calm down, others are trying, unsuccessfully, to solve the problem. Everyone is talking and shouting at the same time and barely something of what is being said can be understood. Amidst the chaos, shouts and insults, Hanak and Beremizer arrive at the scene]

BEREMIZER SAMIR: What is the reason behind all this loud and noise?

OLDEST BROTHER: We are three brothers who have inherited 35 camels from our beloved father who has passed away.

YOUNGEST BROTHER [Looking up while rising both hands]: Peace be upon him!

MIDDLE BROTHER [Looking up while rising both hands]: Peace be upon him!

OLDEST BROTHER [Looking up while rising both hands]: Peace be upon him!

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OLDEST BROTHER: As the oldest of the three, it is my father's wish for me to inherit one half of the camels.

MIDDLE BROTHER: As the middle brother of the three, it is my father's wish for me to inherit one third of the camels.

YOUNGEST BROTHER: As the youngest of the three, it is my father's wish for me to inherit one ninth of the camels.

OLDEST BROTHER [Desperately and hopelessly talking]: But we do not know how to make a fair division and whatever one suggests, the other two disagree! We have tried many solutions, but so far none have been accepted by the three of us! Half of 35 is 17.5, which is not an exact number!

MIDDLE BROTHER [Desperately and hopelessly talking]: One-third of 35 is 11.6, which is not an exact number!

YOUNGEST BROTHER [Desperately and hopelessly talking]: One-ninth of 35 is 3.8, which is also not an exact number!

OLDEST BROTHER [Desperately and hopelessly talking]: How can we make a fair division?

BEREMIZER SAMIR: Cheer up! There is a simple way to make a fair division which I believe will leave you satisfied!

YOUNGEST, MIDDLE AND OLDEST BROTHER [Shout at the same time]: WHAT?? SIMPLE WAY??

OLDEST BROTHER: I see that you did not understand the current situation. And to be honest, I'm not in the mood to explain it again.

BEREMIZER SAMIR: There is no need to repeat it! I have clearly understood! If you allow me to add my friend's camel to the count, I will make a fair division!

[The three brothers gather around, jumping in joy; Hanak turns to Beremizer]

HANAK [Looking mad]: What? Are you crazy? You want to give my camel to these foreigners just to please them?

BEREMIZER SAMIR [Tries to calm Hanak down by placing a hand on his shoulder]: Do not worry, I have a solution.

HANAK [Looking very confused]: This is the only camel that we have! How are we supposed to continue our journey without it? By foot? We will never make it!

BEREMIZER SAMIR: Please, trust me, my friend!

HANAK [Looking confused]: Alright, you win. Take it! I hope you are aware of what you are doing!

[Beremizer turns to the three brothers]

BEREMIZER SAMIR: So, gentleman, do you accept my humble proposal?

YOUNGEST, MIDDLE AND OLDEST BROTHER [At the same time]: “Of course, we accept it.

BEREMIZER SAMIR: So now, there is a total of 36 camels.

[Older Brother walks up to Beremizer]

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BEREMIZER SAMIR: As wished by your father, as the oldest of the three, you were supposed to inherit half of the 35 camels, which was 17.5. With a total of 36 camels, you are now going to inherit 18 camels instead!!

[Oldest Brother steps back, jumping in joy, and Middle Brother gets closet to Beremizer]

BEREMIZER SAMIR: As wished by your father, as the middle brother, you were supposed to inherit one-third of the 35 camels, which was 11.6! With a total of 36 camels, you are now going to inherit 12 camels instead!

[Middle brother steps back, jumping in joy, and Younger Brother gets closer to Beremizer]

BEREMIZER SAMIR: As wished by your father, as the youngest of the three, you were supposed to inherit one-ninth of the 35 camels, which was 3.8! With a total of 36 camels, you are now going to inherit 4 camels instead!

[The Younger Brother steps back and joins the two other brothers; Beremizer turns to the three brothers]

BEREMIZER SAMIR: Congratulations, you have all profited from this division!

[Hanak turns to Beremizer]

HANAK: They have indeed. As for us, we are now on foot!

[Beremizer turns to the three brothers]

BEREMIZER SAMIR: So, let's recap. The oldest brother has received 18 camels, the middle brother 12, and the youngest 4 ...

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[The three brothers, while hugging each other, interrupt Beremizer while exhibiting a smile from ear to ear and jumping in joy]

YOUNGEST, MIDDLE AND OLDEST BROTHER [At the same time]: Yes, exactly!

BEREMIZER SAMIR: ... which amounts to 34 camels.

YOUNGER, MIDDLE AND OLDER BROTHER [At the same time]: What??

BEREMIZER SAMIR: Yes, $18 + 12 + 4 = 34$

[The three brothers look confused by scratching their head and frowning]

BEREMIZER SAMIR: Of the 34 camels, 2 are left. As you know, one of them belonged to my dear friend Hanak. As for the other, for having solved your problem, I believe I am entitled to it.

OLDEST BROTHER: Ok! You are very intelligent, foreigner! You have indeed solved our problem, so you can keep it!

[The three brothers turn to Beremizer and Hanak, show gratitude, say farewell and leave the scene.]

HANAK: Who could have guessed?!

BEREMIZER SAMIR: My friend, you can comfortably ride your camel because now I have one of my own to carry me!

HANAK: Let's continue our journey!

LEARN MORE...

History of Arithmetic

https://en.wikipedia.org/wiki/History_of_arithmetic

What is Arithmetic?

<https://www.cut-the-knot.org/Whats/WhatsArithmetic.shtml>

Online arithmetic exercises on Khan Academy

<https://www.khanacademy.org/math/arithmetic>

Learning basic arithmetic

<https://www.youtube.com/watch?v=TMubSggUOVE>

The Man Who Counted - A Collection of Mathematical Adventures

<https://sparthasarathy.com/ebooks/themanwhocounted.pdf>

Mathematics as History in Theatre: Malba Tahan's – the man who counted

<https://www.youtube.com/watch?v=36hVbkQU3Sw>