PART II: Music & Mathematics

AGE RANGE: 13-15

TOOL 26: BACH AND THE MUSICAL MOEBIUS STRIP

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Educator's Guide

Title: Bach and the musical Moebius strip

Age Range: 13-15 years old

Duration: 1 hour

Mathematical Concepts: Infinity

Artistic Concepts: Two-dimensional vs three-dimensional. Handicraft.

General Objectives: This is a great tool for letting pupils create and at the same time discover a classic image of art.

Instructions and Methodologies: Give the students the possibility to explore math through music and handicraft, by applying it to hands-on doing. This tool is a good basis for your class to discover different math concepts by actually working with their hands.

Resources: This tool provides pictures and videos for the educator to use in the classroom. The topics addressed in these resources will also be an inspiration to find other materials that might be relevant in order to personalize and give nuance to the lesson.

Tips for the educator: Even though there are many hands-on activities involved, remember to be exact about the mathematics.

Desirable Outcomes and Competences: At the end of this tool, the student will be able to: Understand infinity in an improved way.
Explore their handicraft skills.

Debriefing and Evaluation:

Write 3 aspects you liked about this activity:	1. 2. 3.
Write 2 aspects that you have learned	1. 2.
Write 1 aspect for improvement	1.

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Introduction

Is the Moebius band rediscovered by August Ferdinand Moebius or was it Moebius who discovered it? Already the ancient Greeks used the symbol Moebius so carefully studied, to denote eternity and infinity. Moebius, on the other hand, discovered the mathematical properties of the band, that is, it has one side and one edge.

Picture 1

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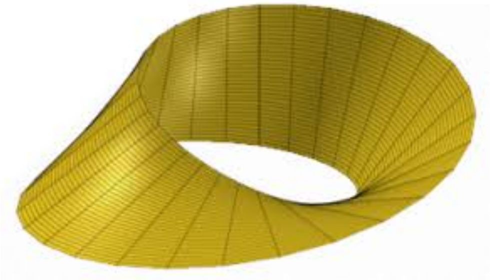
The Moebius strip

August Ferdinand Moebius, born November 17, 1790 in Schulpforta, died September 26, 1868 in Leipzig, was a German mathematician and astronomer.

In 1816, Moebius became extra ordinary professor of astronomy and in 1844 professor of higher mechanics and astronomy at the University of Leipzig. His main research work belongs to the pure mathematics, where he invented a new geometric method, the so-called barycentric calculation. Barycentric calculations use barycentric coordinates .

His most famous result is the so-called Moebius strip, which is a non-orientable surface that only has one side. While Moebius was fully engaged in thinking about how this strip could be used in different ways, at the same time another researcher called Listing was on the same paths about a two-dimensional strip that only has one side and one edge. The two scientists simultaneously published articles about the strip's functions and came to the same result roughly at the same time but it was Moebius name that was finally used to name the strip - and the world now had the Moebius Strip.

The Moebius strip



Picture 2: The Moebius strip https://commons.wikimedia.org/wiki/File:M%C3%B6bius_strip_(plot).png

- is a long rectangular surface
- which is rotated 180 degrees with the ends assembled
- so that along its new path it has one side and one border.

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- The surface is non-orientable and comes back to the same point all the time but mirrored as it only has one side.

Not only Listing and Moebius had been fascinated by the single-sided strip where a dash can be painted on "all" pages without the pen being lifted. Even today, the Moebius strip is used in graphic design as it creates a dynamic and unlimited image. Within science fiction literature, the Moebius strip is used as a description for a possible universe.

The Moebius strip is the mathematical object most used outside the world of mathematics. Comparing the Moebius strip made from paper to a music loop, you will learn that a piece of music which can be played through from the beginning to the end sounding harmonically and melodically correct (basically, sounding nice) is the same as going around the Moebius strip once. Then, if you go through it a second time, yet you start at the end of the piece, so that the last note becomes the first note of the piece, and still it sounds nice, you have Moebius music. To find out this to yourself, print the notes, cut them out, glue them into Moebius strips.

Have a look at a musical Moebius strip here: https://www.youtube.com/watch?v=3x03nJnk-wk

Picture 3: Recycle symbol **Picture 4:** Bracelet

The current use of Moebius strips include, among other things, the conveyor belt that is found in the checkout line in a super market. The actual band that transports the goods

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we buy is shaped like a Möbius strip, as this reduces wear and thus increases the service life. During the early industrialization period, Moebius strip was used as the link between steam engines and the machines that steam engines operated (lathes, threshers, etc.)

Picture 5: Knitted scarf

You can make your own Moebius strip by taking a rectangular paper strip, turning one end half a turn and sticking the ends together. If you now think that someone, say an ant, crawls along the strip, when it crawls, it will be on the other side of the band. Thus, the Moebius band has a single side. Moebius discovered the band as he watched triangulations of the plane.

Glossary

Barycentric coordinates: In astronomy, barycentric coordinates are non-rotating coordinates with the origin at the barycenter of two or more bodies.

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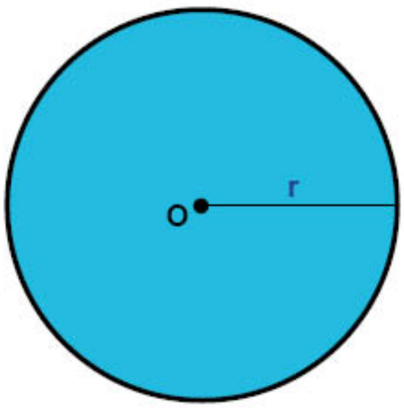
The Math behind Moebius strip

Even though the Moebius strip is not a circle, it deals with the concept of infinity. You could say that the circle and the Moebius strip are equal in that case. Of course you cannot calculate the circumference of a Moebius strip, but there is a connection between the circle and the length of a line.

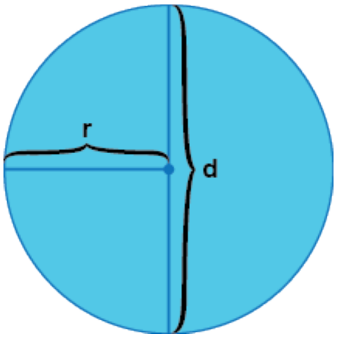
Radius and diameter

A circle is a circular geometric figure starting from a center point. At a certain distance from the center, there is what is sometimes called the circumference of the circle, which is the rounded curve that forms the shape of the circle itself. The distance from the center point to the periphery is called the radius (s) of the circle and is equal to whatever point on the periphery we choose.



Picture 6: Circle

If we have a straight line that goes between two points on the circumference of a circle and fits through the center, we call that distance the diameter of the circle (d). In the figure below, both the radius r and the diameter d are marked.



Picture 7: Circle with diameter and radius

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The diameter of a circle is always twice as long as the radius of the circle:

$$d = 2r$$

Circular circumference and number pi (π)

When examining the perimeter of quadrilaterals and triangles, we came to the conclusion that the perimeter of these figures is equal to the sum of the length of the sides.

But when we study circles, it is not as easy to calculate the perimeter. If we measure the circumference and diameters of different circles, we will soon notice that we get the same ratio every time we divide the circumference of a circle, O, and the diameter of the circle, d.

This ratio is the same for all circles and has the approximate value 3.14159265, when we round the value to eight decimal places. This number is very important in mathematics and is called the number pi, after the Greek letter π. Thus, the ratio between the circumference and diameter of a circle is $\pi \approx 3{,}14$

Using the definition of the number π, we can write a formula for the circumference of a circle, O:

circumference = π ·diameter

$$O = \pi \cdot d$$

Since the diameter d of a circle is always twice as long as the radius r of the circle, we can also write the formula for the circumference of the circle using the radius, like this:

circumference = 2·π· radius

$$O = 2\pi r$$

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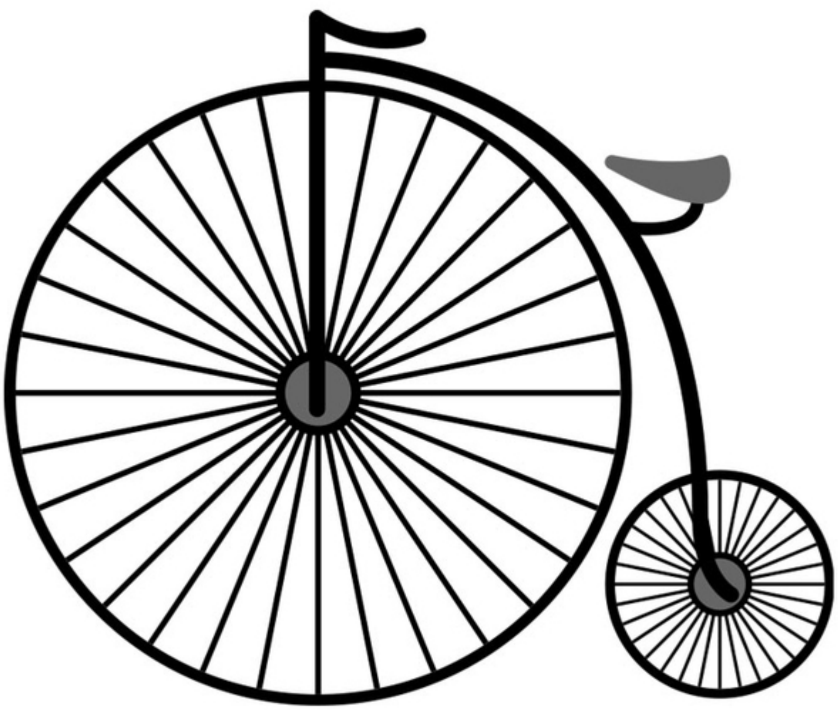
A circumference of a circle is infinite and in the task below you can see the relations between the small and large wheel as an indefinite relation between each other.

TASK

Bicycle

a. How many turns does the rear wheel spin when the front wheel spins one turn. The front wheel diameter is 75 cm. The diameter of the rear wheel is 25 cm.
b. How far will the front wheel go if the rear wheel spins one lap.



Picture 8: Old bicycle

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LEARN MORE…

Ants walking on a Moebius strip:

https://www.youtube.com/watch?v=ZN4TxmWK0bE

Two classroom lessons dealing with the Moebius strip:

https://www.youtube.com/watch?v=JNtKcK27x1s

https://www.youtube.com/watch?v=1xKiSSVY5bI

A short sci-fi movie containing a Moebius strip theme:

https://www.youtube.com/watch?v=HD9MYY0aPug

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