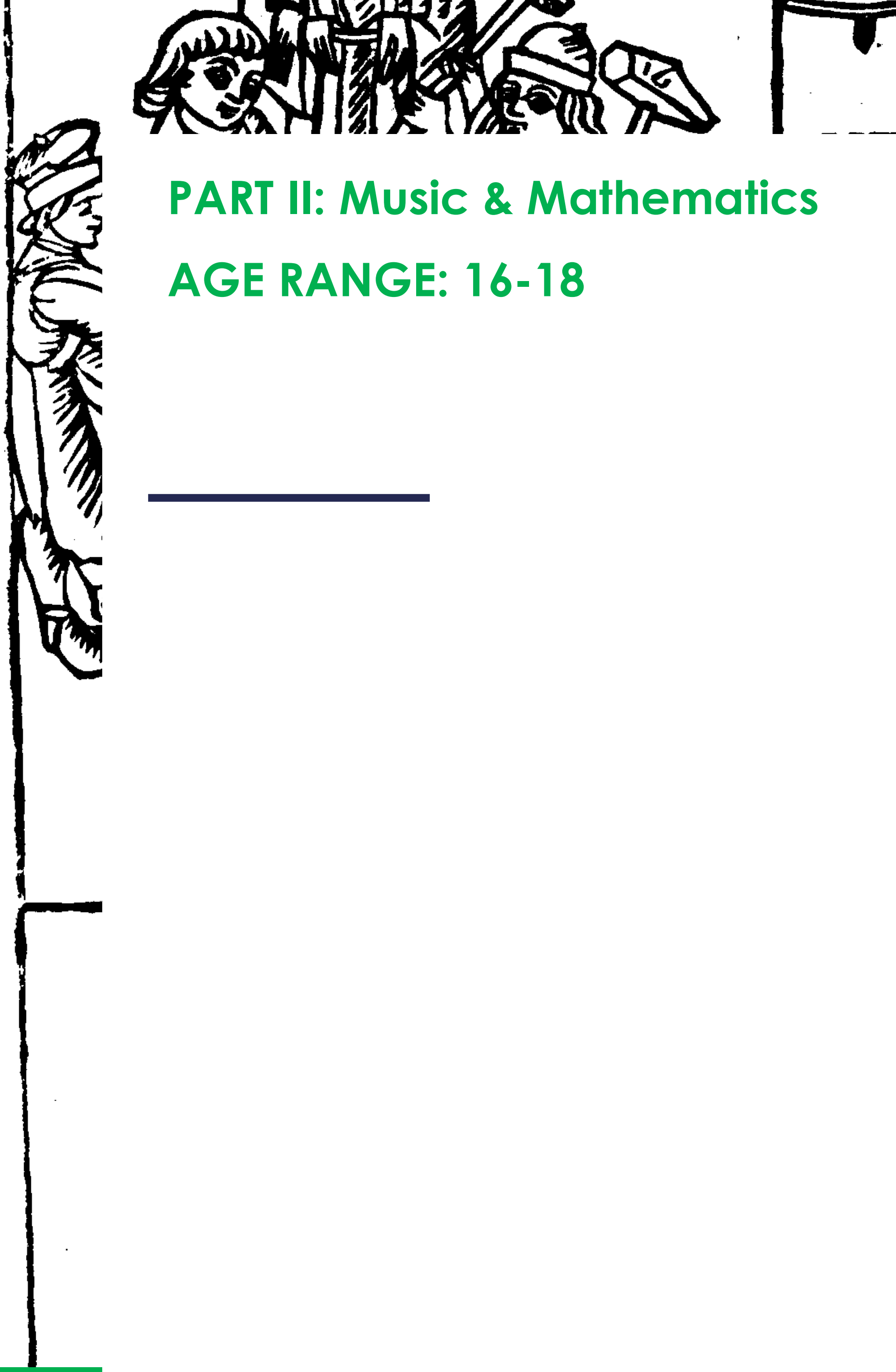
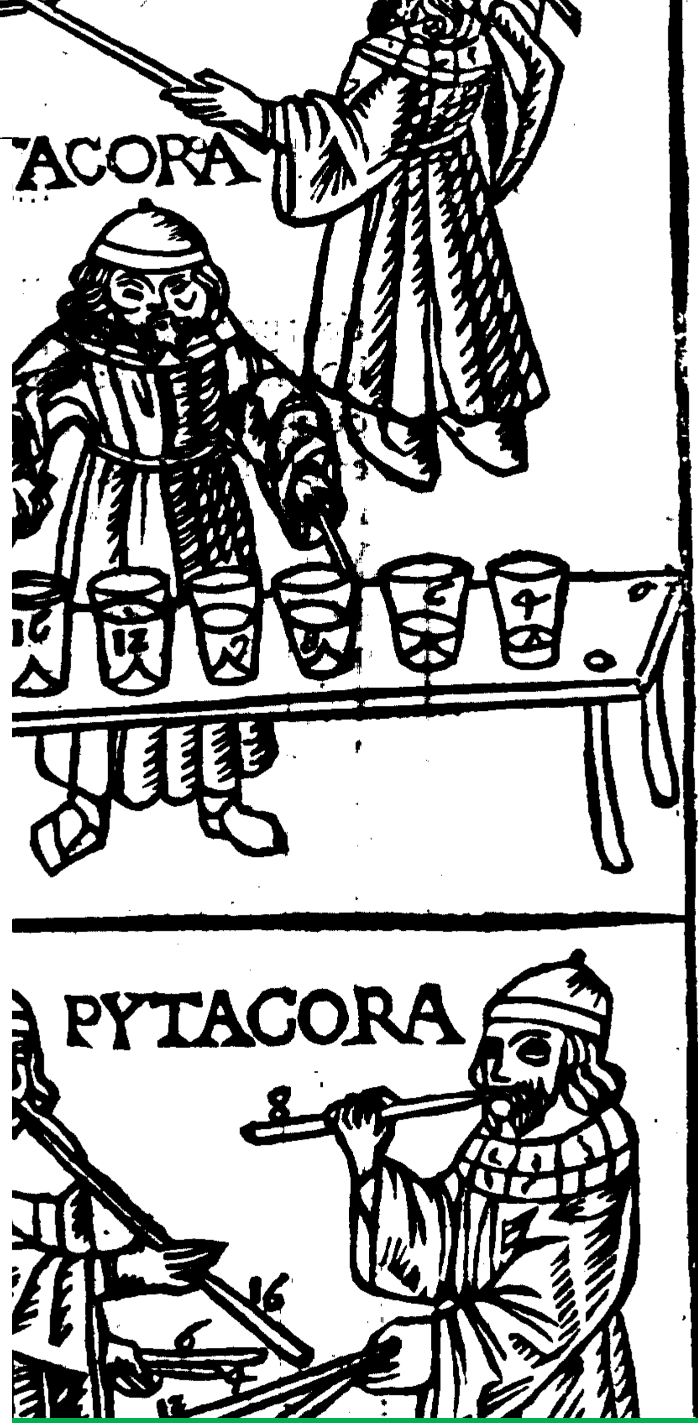




PART II: Music & Mathematics

AGE RANGE: 16-18

TOOL 24: PYTHAGORAS AND HIS MATHEMATICAL MUSIC

Sandgärdskolan

Co-funded by the Erasmus+ Programme of the European Union

Educator's Guide

Title: Pythagoras and his mathematical music

Age range: 16-18

Duration: 2 hours

Mathematical concepts: Equations, algebra, irrational numbers

Artistic concepts: Ancient Greek music, harmony

General objectives: To discover the mathematical concepts hidden in musical compositions and to see that harmony (or what we consider to be harmonious in the western part of the world) can be explained in mathematical terms.

Instructions and methodologies: This tool provides you with ideas for making simple music in class and to make the pupils see that different tones vibrate at different lengths.

Resources: Pictures, glossary, videos

Tips for the educator: Learning by doing has proven to be very efficient, especially with young learners with lower attention span and learning difficulties. Don't forget to always explain what each math concept is useful for, practically.

Learning Outcomes and Competences: At the end of this tool, the student will be able to:

- Understand parts and wholes
- Know the person behind Pythagoras' theorem

Debriefing and Evaluation:

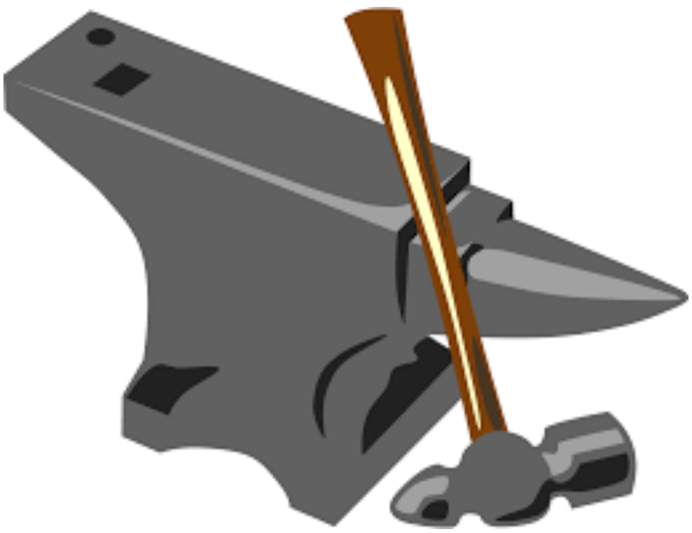
Write 3 aspects you liked about this activity:	1. 2. 3.
Write 2 aspects that you have learned	1. 2.
Write 1 aspect for improvement	1.

This project has been funded with support from the European Commission. This publication reflects the views only of the author, and the Commission cannot be held responsible for any use which may be made of the information contained therein.

Introduction

Picture 1 Pythagoras: https://en.wikipedia.org/wiki/Pythagoras

The Greek philosopher and mathematician Pythagoras, who is said to have lived around 500 BC, is probably most known for his theorem that the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides in a right triangle.



Picture 2 Anvil: https://commons.wikimedia.org/wiki/File:Blacksmith_anvil_hammer.svg

Pythagoras did also discover, however, that musical notes can be described in terms of equations. According to legend he walked into a blacksmith's where he heard the sound of the hammer hitting the anvil. He thought that the tones were harmonious and beautiful at first, but after a while he realized that one of the tones was not. It sounded false.

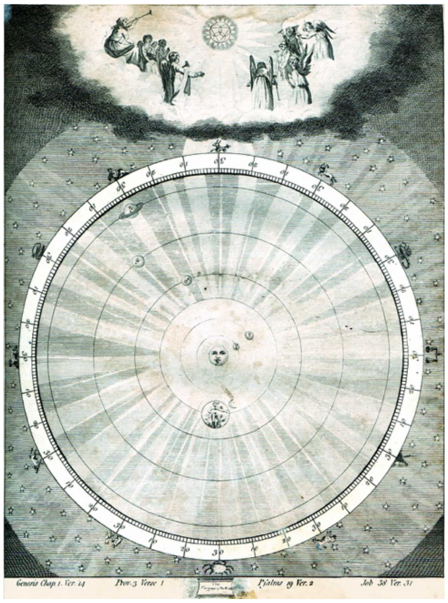
This project has been funded with support from the European Commission. This publication reflects the views only of the author, and the Commission cannot be held responsible for any use which may be made of the information contained therein.

Pythagoras rushed into the shop and started testing the different hammers to see what caused the clear and dissonant tones. He figured out that the only thing that seemed to cause a different tone was the size of the hammer (and not the power of the blow against the anvil or the strength or size of the blacksmith). He took this as evidence for a theory that stated that music could be explained in terms of mathematical equations. It later turned out that he was wrong in terms of hammer and anvil and weight of hammer, but that the theory was correct for string length in one-stringed instruments (https://en.wikipedia.org/wiki/Pythagoras)

This project has been funded with support from the European Commission. This publication reflects the views only of the author, and the Commission cannot be held responsible for any use which may be made of the information contained therein.

Harmony

Pythagoras had a determination that every mathematical concept could be explained in terms of an equation. One of his disciples, Hippasos, tried to find that would determine the square root of 2, but could not and considered this to be a proof that this was an irrational number. Pythagoras was so mad at being proved wrong that he sentenced the poor youngster to death by drowning.



Picture3 Harmony of the world: https://commons.wikimedia.org/wiki/File:Harmonyoftheworld.jpg

Another way of regarding the connection between music and mathematics can be seen in the philosophy that states that the world can be understood as to be in harmony. It is called music of the spheres. It is the sun, moon and the earth that should be in harmony otherwise the order is disrupted. The orbiting bodies produces "music" (in the ancient meaning of it as in musica not necessarily meaning audible music). The Music of the Spheres manifest itself in numbers, visual angles, shapes and sounds – all connected within a pattern of proportion.

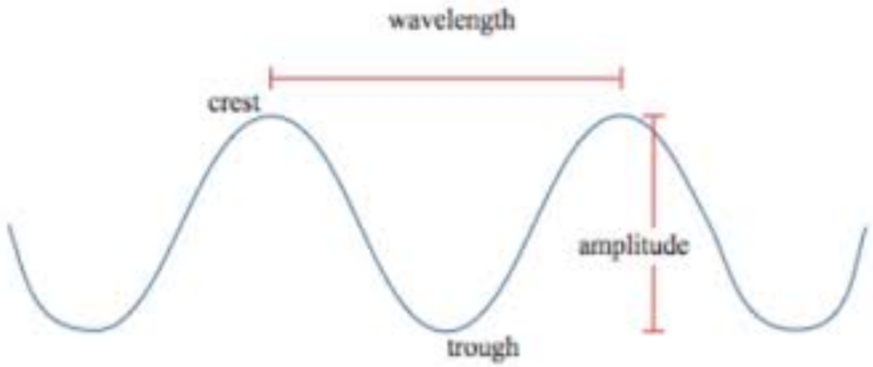
Glossary

Right triangle: A triangle where one of the angles is 90° and the sum of the other two angles is also 90°

This project has been funded with support from the European Commission. This publication reflects the views only of the author, and the Commission cannot be held responsible for any use which may be made of the information contained therein.

The Math behind harmony

The string length theory that Pythagoras was, in a way, on his way to discover can be described in something like this. When you have a string of a certain length and in a certain tense (the power that is used to straighten it) and then take the exact half length of the string, with the same tension you would get a tone that vibrates with double the Hertz (amount of oscillations). Since musical notes can be described in terms of oscillations there is a connection.

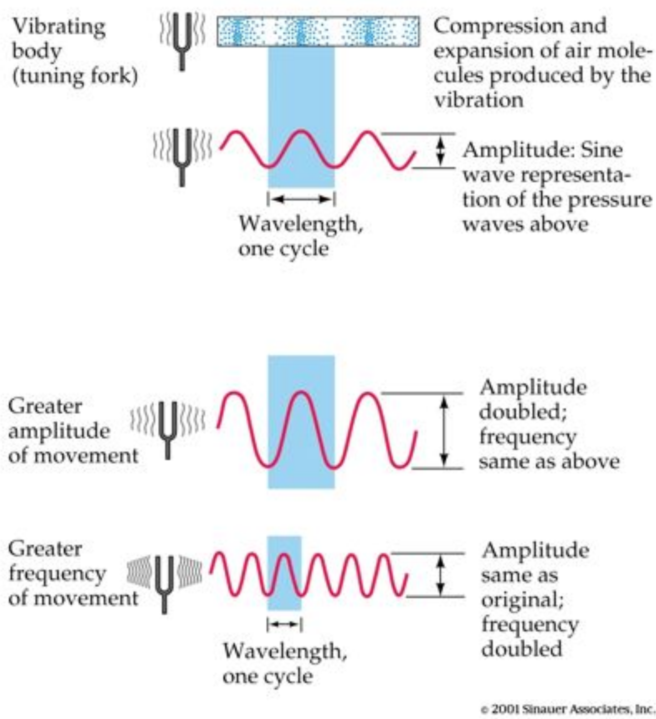


Picture4 Crest through wavelength amplitude
https://commons.wikimedia.org/wiki/File:Crest_trough_wavelength_amplitude.png

The wavelength is the distance between the sound waves in the room where the space waves occurs. The wavelength of a tone in meters is the speed of sound (about 340 meters per second) divided by the frequency in hertz. Frequency is the number of occurrences of a repeating event per unit of time. The lowest audible (for a human being) frequency has a wavelength of 340/15 = 22.7 meters. The highest audible frequency has a wavelength of 340/20000 = 0.017 meters, i.e. 17 millimeters. The air pillar of a wind instrument often has a length corresponding to half the wavelength of its root (lowest possible tone).

This project has been funded with support from the European Commission. This publication reflects the views only of the author, and the Commission cannot be held responsible for any use which may be made of the information contained therein.

Physics of sound



TASKS

Task 1: Create your own music instrument and make use of Pythagoras string length theory.

1. Line up eight glasses or glass bottles on a table in a straight line.
2. Fill them with various amounts of water. For instance 0,25 dl, 0,5 dl, 0,75 dl, 1 dl and so on. If you want to, add some colour to the water
3. Strike each bottle carefully with a small spoon.
4. Arrange the bottles in order from highest to lowest tone. If two of them sound too similar, add or take away some water until you get the sound you want to. If you want to, use a tuning fork to see which tone you have.
5. Create a song to play on your music bottles and invite friends for a concert.

This project has been funded with support from the European Commission. This publication reflects the views only of the author, and the Commission cannot be held responsible for any use which may be made of the information contained therein.

6. What is the connection between tone and amount of water? Do high tones mean short or long wave lengths?

Task 2:

Use the same bottle instrument and compare striking the bottles and blowing in them. Play Twinkle twinkle little star. Try to play it by both blowing in the bottles and by striking them. Why is there a difference?

Task 3:

The wavelength of the tone is 4 times the distance from the mouth to the water surface, a 1/4-part wave is thereby formed in the bottle!

$\frac{340\ m/s}{frequency}$= wavelength

What is the frequency in your bottles? Please note that the tones do not become completely clean due to the shape of the bottle.

LEARN MORE…

You can learn more about Pythagoras and his mathematical philosophy through the Wikipedia sites. Here are a couple of examples:

Pythagoras theorem

Pythagorean tuning

You can also see some videos here:

Pythagorean Tuning [Philosophia Mūsicae: A Philosophy of Music]

The Math of Music - TWO MINUTE MUSIC THEORY #32

The connection between maths and music - Pythagoras Comma (Longer version)

This project has been funded with support from the European Commission. This publication reflects the views only of the author, and the Commission cannot be held responsible for any use which may be made of the information contained therein.