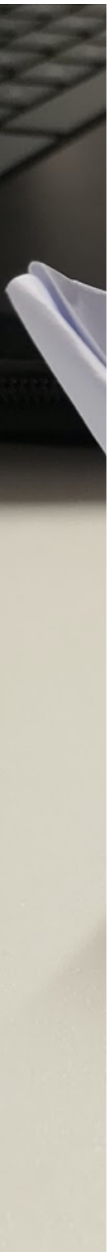
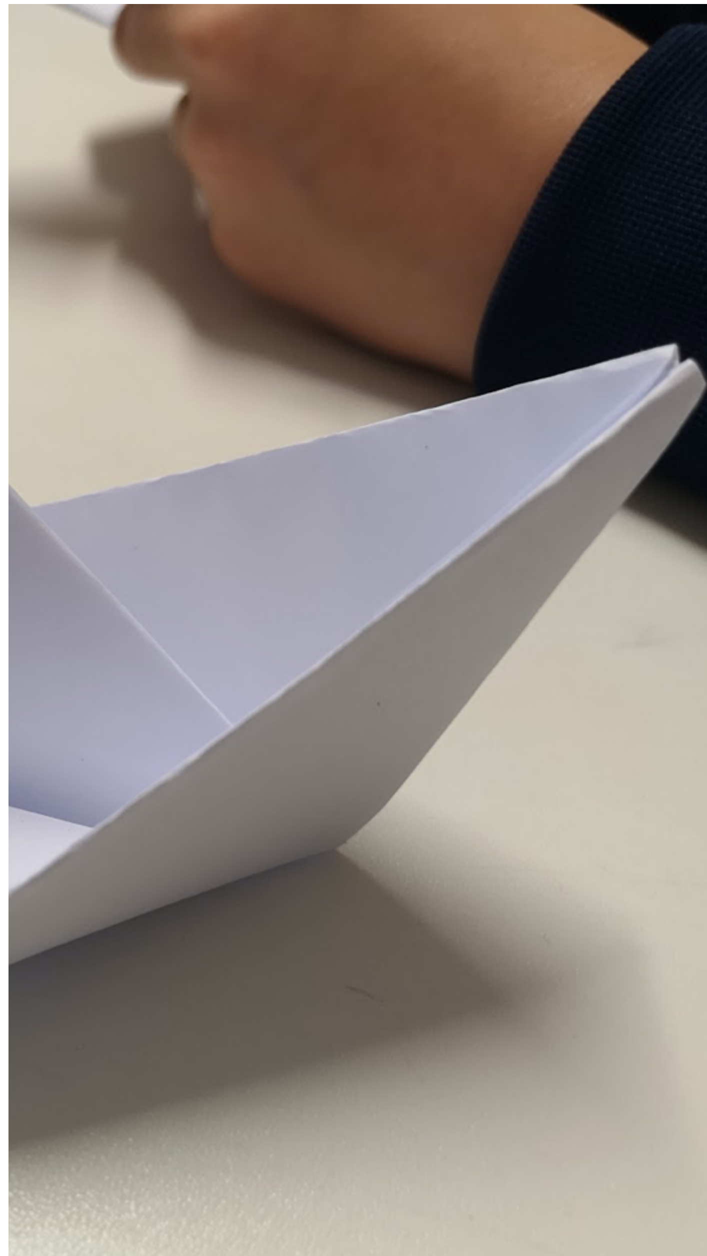
Erasmus+

TheArtOf Maths



PART I: Visual Arts & Mathematics

AGE RANGE: 16-18



TOOL 13: PAPER FOLDING GEOMETRY

Sandgärdskolan

TheArtOf Maths

Co-funded by the Erasmus+ Programme of the European Union

Educator's Guide

Title: Paper folding geometry

Age Range: 16-18 years old

Duration: 2 hours

Mathematical Concepts: dimensions in space, symmetry, polygons, geometric relations, geometric transformations in a plane, cartesian coordinates

Mathematical Concepts: Geometry, trigonometry

Artistic Concepts: Origami

General Objectives: How many times can a paper (model) be flattened without damaging it? Is it possible to solve mathematical equations by folding paper?

Instructions and Methodologies: Give the students the possibility to explore math through origami, by applying it to hands-on doing. This tool is a good basis for your class to discover different math concepts by actually working with their hands.

Resources: This tool provides pictures and videos for you to use in your classroom. The topics addressed in these resources will also be an inspiration for you to find other materials that you might find relevant in order to personalize and give nuance to your lesson.

Tips for the educator: Start with the practical and move towards the mathematical.

Desirable Outcomes and Competences: At the end of this tool, the student will be able to: Understand trigonometry and geometry better

Develop their artistic parts through origami

Debriefing and Evaluation:

Write 3 aspects you liked about this activity:	1. 2. 3.
Write 2 aspects that you have learned	1. 2.
Write 1 aspect for improvement	1.

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Introduction

The name Origami is an old name – Around the beginning of year 600 AC, people learnt the art of folding paper, an art that origins from China. Origami was used very early in Europe and it is uncertain whether there is a connection between China/Japan and Europe. It all started in a mutual tradition by making simple models together. A special paper was preferably used in Japan, the rice paper, but as paper was quite expensive, this was something for the rich. The knowledge of making paper is said to begin at somewhere around 105 AD in China. It started as a hobby for the elite, and mostly belonged to cultural, religious ceremonies. When the art of making paper was spread and well known, paper became cheaper and many more could afford to buy and use it.

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Origami

Ori is Japanese for folding and kami is paper. Paper Folding Geometry is dividing a segment into equal parts through combining Origami and Mathematics. This is not a new idea, an Indian mathematician was during late the 1800's so interested in paper folding to demonstrate proof of geometrical constructions that he studied the use of Origami in the Kindergarten system.

Glossary

Abstraction: is the use of lines, shapes, forms and colors that differ from the accurate depiction of the real world in visual art.

Calligraphy: the art of decorative writing.

Tessellation: is when a shape or image repeats itself infinitely on a plane.

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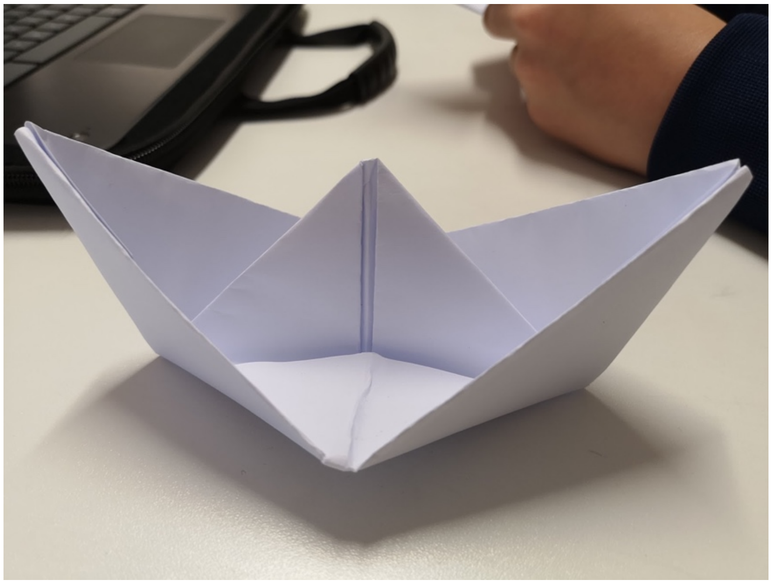
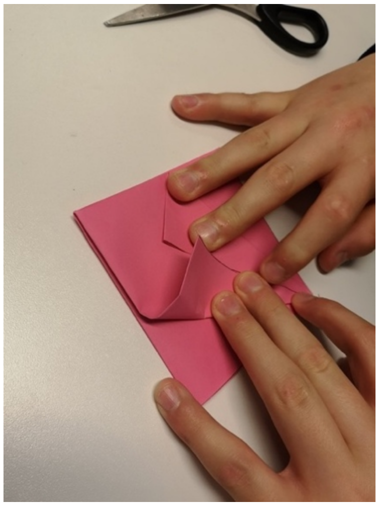
The Math behind Origami

In 1893 T Sundara Rao published his book "Geometric Exercises in Paper Folding". A book which explained approximate trisection of angles and implied constructions of a cube root was possible. This was later, 1930-1990, explained and solved in various ways.



Picture 1

Akira Yoshizawa, born 1911, used Origami to solve geometrical concepts for employees to use at his factory to be able to solve problems related to their work and to make them able to finish their work. He used a "Wet-Folding" Origami technique where water was "showered" upon the paper to make the sharp folding into smooth and more round ones - The Origami items became more sculptured.



Picture 2

Trisecting an arbitrary angle of or doubling the cube has been proven unsolvable using compass and straightened. Paper folding can be constructed to solve equations up to degree 4.

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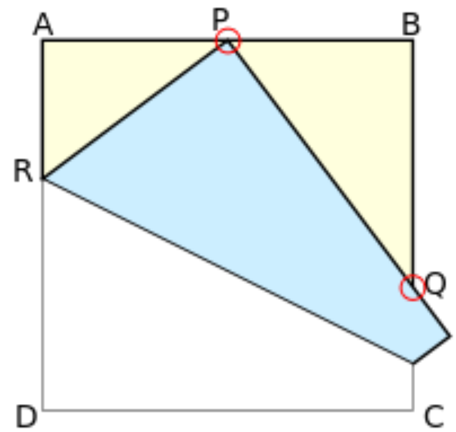
Haga´s theoreme

Surprisingly few folds are necessary to get large odd fractions.



Picture 3

1. First halve a side. Fold corner A and D to B and C, P is now the middle of A and B. Fold it back to a square again.
2. Now fold D to the point P on A-B. We now got three similar triangels, A-P-R, B-Q-P and the small triangle outside the square in the corner C.
3. If the side of the square is 1 and A-R=x, we can now calculate the sides of the A-P-R triangle with Pythagoras:

   $x^2 + (1/2)^2 = (1-x)^2$; $x^2 + ¼ = x^2 -2x +1$; $2x = 1 -¼$; $x=3/8$.

4. Because the triangles are similar we can now calculate the length of B-Q, let us call it "L".

   $L/(1/2)=(1/2)/(3/8)$; $L=2/3$!

5. Now we fold the corners B and A so B reaches Q, and we divide B-Q in the middle and so produce 1/3.

If we now repeat step 2-5, but instead of folding D to P fold corner A to the new middle on B-Q, we can produce ¼, 1/5 and so on!

For more explanations and pictures, visit "Folding fractions"!

From this we can learn that Origami links with Geometry. Creases and edges represent lines, intersections themselves, represent points. Because of its manipulative and experimental nature, Origami could become an effective context for the

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learning and teaching of Geometry. It leads the student to study the effects of the folding and seek patterns.

The manipulative nature of Origami allows much experimenting, comparing, visualizing, discovering and conjecturing.

The future and Origami

Already today, simple folding is applied in, among other things, solar panels and antennas used in space missions. There are examples of panels folded together like accordion or an umbrella.

One of NASA's ongoing projects concerns a large expandable screen similar to a sunflower. As it unfolds into space, it should block the light from distant stars. This will allow a space telescope to take pictures of other planets in the solar system. Nasa is also developing a solar panel that has a diameter of 2.7 meters folded. When the solar panel is unfolded, it should have a diameter of 25 meters. The solar panel should capture solar energy and deliver the energy to the earth.

Take less space

The advantage of using origami is that you can pack flat structures that you want to unfold in a better way. In that way, it takes less space than it would if you had a simpler folding as for today - That it takes a little space is very important in the space industry because there is little space in the space capsule that is sent out into space. One challenge is that the material used is thicker than paper and thus increases the thickness of each fold.

Origami technology

is particularly useful for spacecraft applications that need to open from the center outward, like a flower.

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Despite the new plans, it will probably take a long time before origami becomes common in space technology as new technology and design takes a very long time before it becomes the new way of building, as long as the old way works.

The Golden Ratio + Origami?

Absolutely! How to make a rectangle by folding papers using Pythagoras Theorem? Watch the movie below...

https://www.youtube.com/watch?v=E6ioUH5tcbM

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TASK

Platonic bodies

Material

Clip sheet with the platonic bodies. Scissors, glue, tape and colored pens. Enlarge the clip sheet to A3; it will be easier to fold and build them.

Before you start

Start with task A-C. There are only 5 platonic bodies . The side surfaces of the bodies are regular and the angle sum has to be less than 360° to make it possible for a corner of the body.

History about the Platonic bodies

The platonic bodies have got their names from Platon. Platon (430-349 B.C) was a greek philosopher and disciple to Aristoteles.

The platonic bodies are regular polyhedrons. There are bodies where all sides are same regular polygons. These bodies symbolized for the thinkers of the time, the four elements; fire, air, earth and water. The tetrahedral symbolise fire, the hexahedron symbolize
earth, the octahedron symbolize water, the icosahedron symbolize air. The dodecahedron symbolize The universe.

Task

You shall investigate different geometrical bodies. See similarities and differencies. Then fill in the chart below.

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Body	Number of sides	Shape on the side surfaces	Number of degrees in the side surface corner	The angle sum in the corners of the body
tetrahedral	4	equilateral triangle	60	3*60=180
hexahedron				
octahedron				
dodecahedron				
icosahedron				

A Color, cut and build the platonic bodies.

B There are only five platonic bodies. Why can't we construct one more?

Clue: Look in the chart and the column With angle sum in the corners of the body.

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LEARN MORE…

The Math and magic of origami
https://www.ted.com/talks/robert_lang_folds_way_new_origami

Origami folding blindfolded
https://www.ted.com/talks/bruno_bowden_rufus_cappadocia_watch_me_fold_origami_blindfolded

Mathematical origami
https://mathigon.org/origami

The passion behind origami
https://global.honda/70th-anniv/origami.html

Origami art projects for kids
https://www.artforkidshub.com/origami/

Easy origami instructions and diagrams
https://www.origamiway.com/easy-origami.shtml

Origami patterns
https://www.worldwildlife.org/pages/origami-patterns

+Plus Magazine, Folding fractions
https://plus.maths.org/content/folding-numbers

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