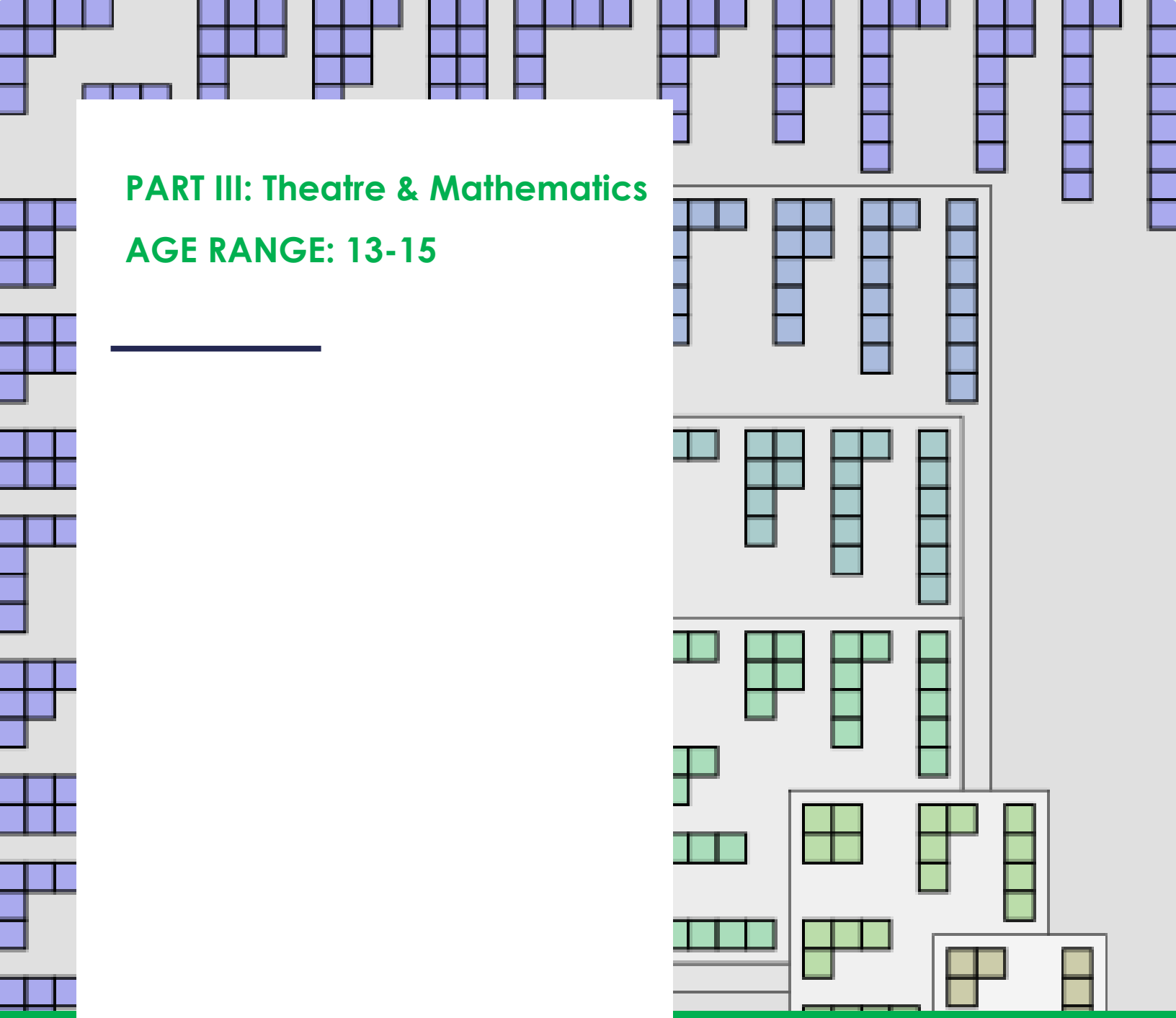

PART III: Theatre & Mathematics

AGE RANGE: 13-15

TOOL 29: GEOMETRY THROUGH EUCLID'S ELEMENTS

C.I.P. Citizens In Power

TheArtOf Maths

Co-funded by the Erasmus+ Programme of the European Union

Educator's Guide

Title: Geometry through Euclid's 'Elements' (school play)

Age Range: 13-15 years old

Duration: 2 hours

Mathematical Concepts: Mathematical Elements, Theorem, Straight line, Mathematical point

Artistic Concepts: Drama/play.

General Objectives: Mathematical wise, students will familiarize with the father of geometry Euclid's and his work thus getting acquaintance with some of the mathematical concepts he dealt with. Literacy wise, they will locate some concepts of the Classical Era and generally the historical period like the fatherhood of the documents and meet another historical feature which is Hypatia.

Instructions and Methodologies: The teacher is the stage director. Considering that the terminologies 'Mathematical Elements, Theorem, Straight line' are already known through the school syllabus, this activity can refresh their memory and enhance their establishment. For acting on the play, the classroom should be divided into groups of six, given that these are the participants of the play. Three have a main role and three only one small line, so this is appropriate even for students who are shy or not so willing to participate. The act can be made in a form of a contest between groups and the winning team can get a reward.

Resources: This tool provides pictures and videos, references and extra material to be used in the classroom.

Tips for the educator: There is the flexibility to act according to the timeframe and exact age of students. In the tasks you are given several alternatives such as, giving information on the students a priori about Euclid's and his 'Elements' or asking from the students to retrieve these information themselves. There is also another alternative to ask students to learn the script by heart or to just have a quick read in their groups and then present it. It is also up to the educator, how the 'competition' and 'voting' will proceed and if there is going to be a reward.

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Desirable Outcomes and Competences:

- Students will learn to:
  - unlock and be more open through acting,
  - improve their memorization during rehearsal and
  - learn basic mathematical terms;
  - describe historical features,
  - remember basic mathematical terms such as the theorem,
  - familiarize with geometry,
  - familiarize with the terminology of the straight line before and after Euclid's influence.

Debriefing Exercise:

You can use these cards sometimes called EXIT CARDS either by a hard copy made from before or simply by posing these statements on board and the students write their answers on a paper which they will leave preferably anonymously while exiting the room. The specific formative strategy is called 3,2,1. For more strategies you can visit:

https://www.bhamcityschools.org/cms/lib/AL01001646/Centricity/Domain/131/70%20Formative%20Assessments.pdf

3-2-1	
Write 3 things you liked about this activity	**1.** **2.** **3.**
Write 2 things you have learned	**1.** **2.**
Write 1 aspect for improvement	**1.**

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Introduction

Mathematics can be found in several arts e.g. literature, cinematography, drama. In drama their presence is noteworthy in theatrical texts and dramatized dialogues. Learning by acting has proven to be very efficient, especially with young learners with lower attention span and learning difficulties. It is also a desirable way to construct a positive attitude towards mathematics. When used correctly it can enhance the receptive ability of students, the development of the collectivity in learning and increase of the participation of the whole class.

Wither these theatrical texts have been especially made for school plays or are classical theatrical documents, their learning outcomes can be beneficiary for students as an alternative and fun way of learning. Recently a European project called MATHeatre aiming to encourage students to stimulate their imagination and express mathematical ideas using theatrical skills; has found that there are several benefits to dramatizing with a mathematical context. It is accepted that through a theatrical activity with a mathematical content, students not only can familiarize themselves and ultimately learn mathematical concepts by heart, but also there are other benefits with this approach whilst performing in public such as improving their self esteem, learning to cooperate, fluidize speech and improving attention, concentration and listening to each other. More specifically a theatrical act can be used to discover a new concept or reinvent a new concept. Here it will be used after studying the concept following the theory and the exercises.

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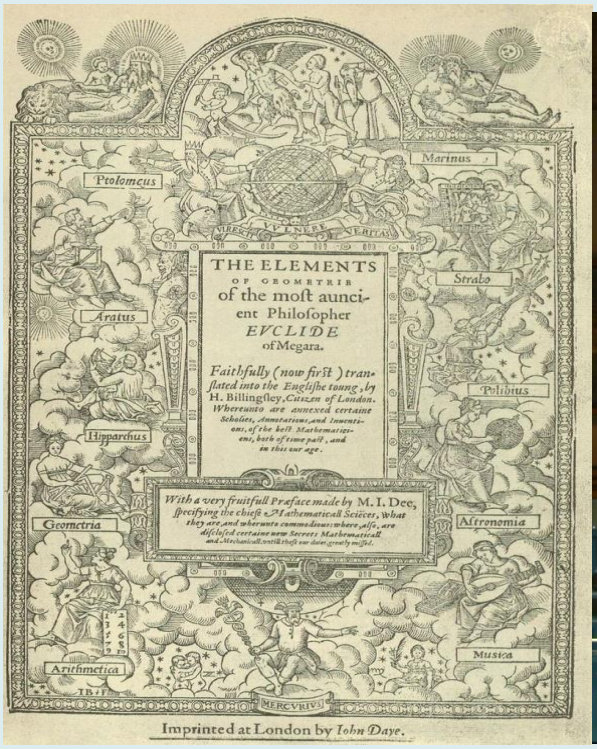
Literary Approach

Since ancient times great mathematicians have used public speaking to communicate their knowledge to the public. One of them, Euclid of Alexandria, a Greek mathematician (300BC), is considered to be the founder or father of geometry. His 'Elements' is one of the most influential works in the history of mathematics, serving as the main textbook for teaching mathematics (especially geometry) from the time of its publication until the late 19th or early 20th century. 'The Elements' is a mathematical treatise consisting of 13 books. In the Elements, Euclid deduced the theorems of what is now called Euclidean geometry from a small set of axioms.



Picture 1: The *Elements,* a series of 13 books written by Euclid [1]

Picture 2: 19th-century statue of Euclid by Joseph Durham in the Oxford University Museum of Natural History [2]

[1] (Retrieved from: https://el.wikipedia.org/wiki/%CE%A3%CF%84%CE%BF%CE%B9%CF%87%CE%B5%CE%AF%CE%B1)
[2] (Retrieved from: https://en.wikipedia.org/wiki/Euclid)

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Euclid's Elements is by far the most famous mathematical work of classical antiquity, and also has the distinction of being the world's oldest continuously used mathematical textbook. Little is known about the author, beyond the fact that he lived in Alexandria around 300 BCE. The main subjects of the work are geometry, proportion, and number theory. Most of the theorems appearing in the Elements were not discovered by Euclid himself, but were the work of earlier Greek mathematicians such as Pythagoras (and his school), Hippocrates of Chios, The Aetius of Athens, and Eudoxus of Cnidos. However, Euclid is generally credited with arranging these theorems in a logical manner, so as to demonstrate (admittedly, not always with the rigor demanded by modern mathematics) that they necessarily follow from five simple axioms. Euclid is also credited with devising a number of particularly ingenious proofs of previously discovered theorems: e.g., Theorem 48 in Book 1.The geometrical constructions employed in the Elements are restricted to those which can be achieved using a straight-rule and a compass. Furthermore, empirical proofs by means of measurement are strictly forbidden: i.e.,any comparison of two magnitudes is restricted to saying that the magnitudes are either equal, or that one is greater than the other.

The Elements consist of thirteen books:

Book 1 outlines the fundamental propositions of plane geometry, including the three cases in which triangles are congruent, various theorems involving parallel lines, the theorem regarding the sum of the angles in a triangle, and the Pythagorean theorem.

Book 2 is commonly said to deal with "geometric algebra", since most of the theorems contained within it have simple algebraic interpretations.

Book 3 investigates circles and their properties, and includes theorems on tangents and inscribed angles.

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Book 4 is concerned with regular polygons inscribed in, and circumscribed around, circles. Book 5 develops the arithmetic theory of proportion.

Book 6 applies the theory of proportion to plane geometry, and contains theorems on similar figures.

Book 7 deals with elementary number theory: e.g., prime numbers, greatest common denominators, etc.

Book 8 is concerned with geometric series.

Book 9 contains various applications of results in the previous two books, and includes theorems on the infinitude of prime numbers, as well as the sum of a geometric series.

Book 10 attempts to classify incommensurable (i.e., irrational) magnitudes using the so-called "method of exhaustion", an ancient precursor to integration.

Book 11 deals with the fundamental propositions of three-dimensional geometry.

Book 12 calculates the relative volumes of cones, pyramids, cylinders, and spheres using the method of exhaustion. Finally,

Book 13 investigates the five so-called Platonic solids.

This edition of Euclid's Elements presents the definitive Greek text—i.e., that edited by J.L. Heiberg (1883–1885)—accompanied by a modern English translation, as well as a Greek-English lexicon. Neither the spurious books 14 and 15, nor the extensive scholia which have been added to the Elements over the centuries, are included.

(Translation retrieved from: http://farside.ph.utexas.edu/Books/Euclid/Elements.pdf).

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Glossary

Euclid: Euclid is the anglicized version of the Greek name Εὐκλείδης, which means "renowned, glorious"

Elements: is a mathematical treatise consisting of 13 books attributed to the ancient Greek mathematician Euclid in Alexandria, Ptolemaic Egypt c. 300 BC. It is a collection of definitions, postulates, propositions (theorems and constructions), and mathematical proofs of the propositions. The books cover plane and solid Euclidean geometry, elementary number theory, and incommensurable lines. Elements is the oldest extant large-scale deductive treatment of mathematics. It has proven instrumental in the development of logic and modern science, and its logical rigor was not surpassed until the 19th century.

Hypatia: Hypatia[a] (born c. 350–370; died 415 AD) was a Hellenistic Neoplatonist philosopher, astronomer, and mathematician, who lived in Alexandria, Egypt, then part of the Eastern Roman Empire. She was a prominent thinker of the Neoplatonic school in Alexandria where she taught philosophy and astronomy. She is the first female mathematician whose life is reasonably well recorded.

Milan Kundera: Born 1st of April 1929, he is a Czech-born French writer who went into exile in France in 1975, and became a naturalised French citizen in 1981. He "sees himself as a French writer and insists his work should be studied as French literature and classified as such in book stores"

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THE SCRIPT

Translated from Greek by the Project's Team. The author of the play 'The children of Euclid's' is Elias Konstantopoulos, the fragment is an adaption made by Elias Kerasides for a school play in 2000-2001 retrieved from: http://www.hdml.gr/pdfs/conferences/222.pdf?fbclid=IwAR2Duji3A-JIESVpSAvmpDlM768JCTZMRTeDuB55r1_FVRqgVPsJ6ES3N3Y .

Euclid:	**I am Euclid. I was born in 330 BC and I am still alive. I have lived with my ruler and compass. Whilst I am thinking that with this compass, I was just trying to make straight lines whilst some teachers were using it to hit students…. It has also changed because mine was making only circles, it was not used to measure straight sections. But let's be it, there are worst things in the world.**
The Voice:	**We would like to see your Curriculum Vitae.**
Euclid:	**Why? Will you hire me as your employ? Anyhow. I was born in Alexandria, I studied in Athens and then I returned to Alexandria, as the principle of the Museum when Ptolemy I was the king. There I wrote the *Elements*.**
The Voice:	**What was the Museum?**
Euclid:	**It was the University of Alexandria. Along with the library it was for many centuries the center of the whole world.**
The Voice:	**And what are the *Elements*?**
Euclid:	**The *Elements* are my children. I was blessed by God to have 13, not all mine by blood. Some are adopted, but to me all beloved, and wonderful of course.**
The Voice:	**So, before you there was no geometry? Why didn't you write the history of geometry?**
Euclid:	**I haven't anticipated. I haven't anticipated. But I am going to do it soon.**
The Voice:	**So let's see what you have written.**

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Euclid:	All together they include 131 definitions and 465 propositions. Let us begin with my first book.
Hypatia:	I include 23 definitions, 5 postulates and 5 common notions and 48 propositions.
Euclid:	Take it a bit slower. Do not overwhelm them with too much information or you will scare them.
The Voice:	Hypatia? Why have you named it like that? Who was Hypatia?
Euclid:	The first and last female mathematician of ancient times. This is one of my real children, since not only she lived after me but her father, Theonas rescued my *Elements*. The version that you have comes from him.
Hypatia:	I am confused now. Am I your child or Theonas?
Euclid:	You are my spiritual child.
The Voice:	OK, stop arguing. Let's cut the chase. Definitions, ok we can imagine what they are about, but what about postulates?
Hypatia:	Well, yes, the definitions are about geometrical shapes for example the straight line, the corner of the triangle, the triangle the circle.
Euclid:	So tell us the definition of the straight line.
Hypatia:	The straight line is that which is equally extended between its points.
The Voice:	Smart but doesn't seem quite right.
Euclid:	Well, if you do not like my definition can you give me one of your own?
The Voice:	The orbit of an airplane….
Voice 1:	A bright beam…
Voice 2:	The fastest route….
Voice 3:	Our thoughts when they fly…
Euclid:	All of your thoughts are drawn by gravity. Mine is free. Like *The Unbearable Lightness of Being*
The Voice:	I see you have read Kundera, but tell us, what is a straight line?
Euclid:	"Breadthless length" which "lies equally with respect to the points on itself"; and I know what you are about to ask now. What is a point? You

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	believe that these notions cannot be defined, but definitions help us understand them better.
The Voice:	**But how can we from a spot without any dimensions create a line with certain dimensions how can anyone from something 'infinitely small' extract something 'infinitely large'?**
Hypatia:	**Do not consider the spot as a material entity. Neither the line is something material. The line in geometry exists only in our minds, what we draw as a line is just its ghost.**
The Voice:	**You almost persuaded us about the definitions, but what about the postulates?**
Hypatia:	**Postulates are initial assumptions about a certain filed, in our case, Geometry.**

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Definitions

Geometry: the part of mathematics concerned with the size, shape and relative position of figures, or the study of lines, angles, shapes and their properties

Theorem: In mathematics, a theorem is a statement that has been proven on the basis of previously established statements, such as other theorems, and generally accepted statements, such as axioms.

Postulates: A statement, also known as an axiom, which is taken to be true without proof. Postulates are the basic structure from which lemmas and theorems are derived. The whole of Euclidean geometry, for example, is based on five postulates known as Euclid's postulates.

Line: A line is a straight one-dimensional figure having no thickness and extending infinitely in both directions. A line is sometimes called a straight line or, more archaically, a right line (Casey 1893), to emphasize that it has no "wiggles" anywhere along its length. While lines are intrinsically one-dimensional objects, they may be embedded in higher dimensional spaces. Harary (1994) called an edge of a graph a "line."

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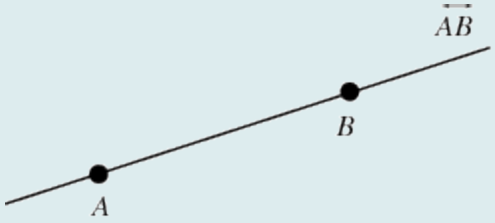

A line is uniquely determined by two points, and the line passing through points A and B is denoted $\overleftrightarrow{AB}$. Similarly, the length of the finite line segment terminating at these points may be denoted $\overline{AB}$. A line may also be denoted with a single lower-case letter (Jurgensen et al. 1963, p. 22).

Euclid defined a line as a "breadthless length," and a straight line as a line that "lies evenly with the points on itself" (Kline 1956, Dunham 1990).

Consider first lines in a two-dimensional plane. Two lines lying in the same plane that do not intersect one another are said to be parallel lines. Two lines lying in different planes that do not intersect one another are said to be skew lines.

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TASK

- Divide in groups of six to act in this play either by having the exact words or by memorizing the text depending on the instructions you have been given. There are 3 protagonists and 3 sturgeons.

- Music: it is always welcome as a background.
- Accessories: You can use some mathematical instruments from the classroom such as a ruler or a compass for being more creative and theatrical.
  Costumes: They can be as simple as a shirt to be used as a gown instructed a priori to be brought from home.

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LEARN MORE...

If you want to further investigate on the topics addressed in this tool, you may go through the following links:

A study on the Role of Drama in Learning Mathematics:
https://www.researchgate.net/publication/274582627_A_Study_on_the_Role_of_Drama_in_Learning_Mathematics

Project Learning mathematics through new communication factors:
http://www.le-math.eu/assets/files/MATHeatre-Guidelines-V1-September2013.pdf

Mathematics Glossary:
https://www.storyofmathematics.com/glossary.html#G

http://www.hdml.gr/pdfs/conferences/222.pdf?fbclid=IwAR19Ravx1wXFJ2h8VnVva0ZSLgBH5OjNeZy7APRpttHKTW59TBr32Xhkjjo

Article: Using drama to teach pupils math's concepts
https://www.straitstimes.com/singapore/education/using-drama-to-teach-pupils-maths-concepts

Mathematics Resource:
http://mathworld.wolfram.com/Line.html

Euclid's Elements of Geometry English translation, by Richard Fitzpatrick
http://farside.ph.utexas.edu/Books/Euclid/Elements.pdf

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